

General Certificate of Education  
June 2008  
Advanced Level Examination



**MATHEMATICS**  
**Unit Pure Core 3**

**MPC3**

Friday 23 May 2008 9.00 am to 10.30 am

**For this paper you must have:**

- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Answer **all** questions.

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1 Find  $\frac{dy}{dx}$  when:

(a)  $y = (3x + 1)^5$ ; *(2 marks)*

(b)  $y = \ln(3x + 1)$ ; *(2 marks)*

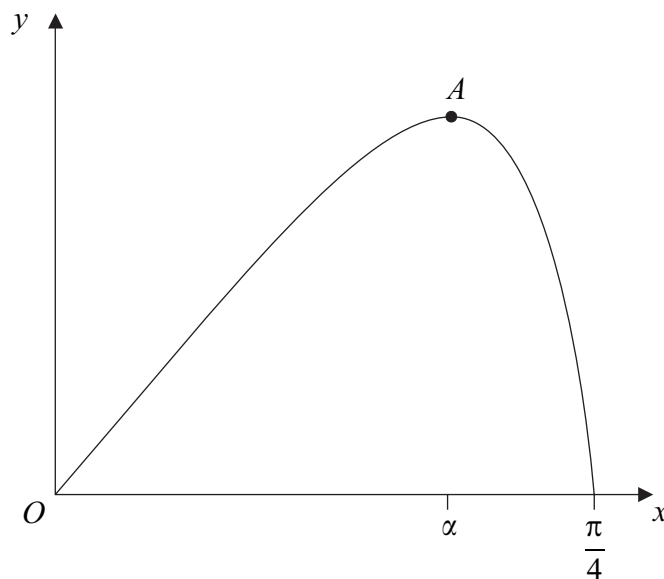
(c)  $y = (3x + 1)^5 \ln(3x + 1)$ . *(3 marks)*

2 (a) Solve the equation  $\sec x = 3$ , giving the values of  $x$  in radians to two decimal places in the interval  $0 \leq x < 2\pi$ . *(3 marks)*

(b) Show that the equation  $\tan^2 x = 2 \sec x + 2$  can be written as  $\sec^2 x - 2 \sec x - 3 = 0$ . *(2 marks)*

(c) Solve the equation  $\tan^2 x = 2 \sec x + 2$ , giving the values of  $x$  in radians to two decimal places in the interval  $0 \leq x < 2\pi$ . *(4 marks)*

- 3 A curve is defined for  $0 \leq x \leq \frac{\pi}{4}$  by the equation  $y = x \cos 2x$ , and is sketched below.



- (a) Find  $\frac{dy}{dx}$ . (2 marks)
- (b) The point  $A$ , where  $x = \alpha$ , on the curve is a stationary point.
- (i) Show that  $1 - 2\alpha \tan 2\alpha = 0$ . (2 marks)
- (ii) Show that  $0.4 < \alpha < 0.5$ . (2 marks)
- (iii) Show that the equation  $1 - 2x \tan 2x = 0$  can be rearranged to become  $x = \frac{1}{2} \tan^{-1} \left( \frac{1}{2x} \right)$ . (1 mark)
- (iv) Use the iteration  $x_{n+1} = \frac{1}{2} \tan^{-1} \left( \frac{1}{2x_n} \right)$  with  $x_1 = 0.4$  to find  $x_3$ , giving your answer to two significant figures. (2 marks)
- (c) Use integration by parts to find  $\int_0^{0.5} x \cos 2x \, dx$ , giving your answer to three significant figures. (5 marks)

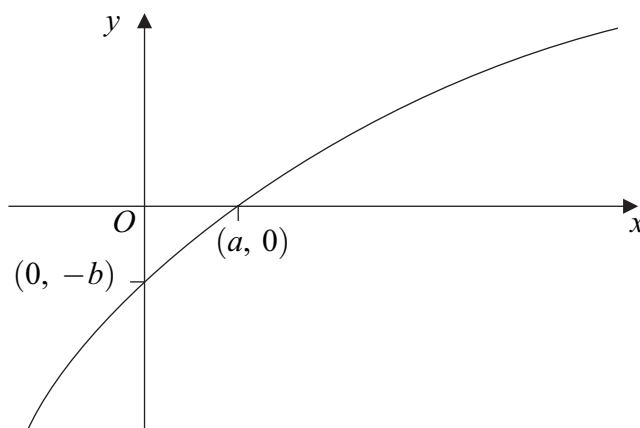
4 The functions  $f$  and  $g$  are defined with their respective domains by

$$f(x) = x^2, \quad \text{for all real values of } x$$

$$g(x) = \frac{1}{2x-3}, \quad \text{for real values of } x, \quad x \neq \frac{3}{2}$$

- (a) State the range of  $f$ . (1 mark)
- (b) (i) The inverse of  $g$  is  $g^{-1}$ . Find  $g^{-1}(x)$ . (3 marks)
- (ii) State the range of  $g^{-1}$ . (1 mark)
- (c) Solve the equation  $fg(x) = 9$ . (3 marks)

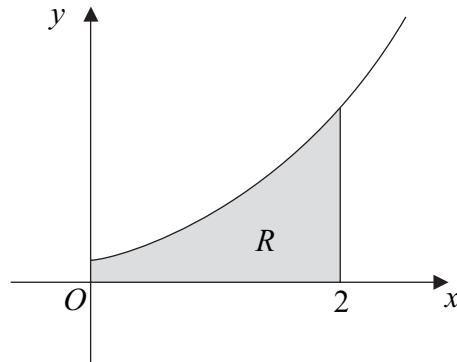
5 (a) The diagram shows part of the curve with equation  $y = f(x)$ . The curve crosses the  $x$ -axis at the point  $(a, 0)$  and the  $y$ -axis at the point  $(0, -b)$ .



On separate diagrams, sketch the curves with the following equations. On each diagram, indicate, in terms of  $a$  or  $b$ , the coordinates of the points where the curve crosses the coordinate axes.

- (i)  $y = |f(x)|$ . (2 marks)
- (ii)  $y = 2f(x)$ . (2 marks)
- (b) (i) Describe a sequence of geometrical transformations that maps the graph of  $y = \ln x$  onto the graph of  $y = 4 \ln(x + 1) - 2$ . (6 marks)
- (ii) Find the exact values of the coordinates of the points where the graph of  $y = 4 \ln(x + 1) - 2$  crosses the coordinate axes. (4 marks)

- 6 The diagram shows the curve with equation  $y = (e^{3x} + 1)^{\frac{1}{2}}$  for  $x \geq 0$ .



- (a) Find the gradient of the curve  $y = (e^{3x} + 1)^{\frac{1}{2}}$  at the point where  $x = \ln 2$ . (5 marks)
- (b) Use the mid-ordinate rule with four strips to find an estimate for  $\int_0^2 (e^{3x} + 1)^{\frac{1}{2}} dx$ , giving your answer to three significant figures. (4 marks)
- (c) The shaded region  $R$  is bounded by the curve, the lines  $x = 0$ ,  $x = 2$  and the  $x$ -axis.

Find the exact value of the volume of the solid generated when the region  $R$  is rotated through  $360^\circ$  about the  $x$ -axis. (4 marks)

- 7 (a) Given that  $y = \frac{\sin \theta}{\cos \theta}$ , use the quotient rule to show that  $\frac{dy}{d\theta} = \sec^2 \theta$ . (3 marks)
- (b) Given that  $x = \sin \theta$ , show that  $\frac{x}{\sqrt{1-x^2}} = \tan \theta$ . (2 marks)
- (c) Use the substitution  $x = \sin \theta$  to find  $\int \frac{1}{(1-x^2)^{\frac{3}{2}}} dx$ , giving your answer in terms of  $x$ . (5 marks)

**END OF QUESTIONS**

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