

General Certificate of Education  
June 2007  
Advanced Subsidiary Examination



**MATHEMATICS**  
**Unit Pure Core 2**

**MPC2**

Monday 21 May 2007 9.00 am to 10.30 am

**For this paper you must have:**

- an 8-page answer book
  - the **blue** AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Answer **all** questions.

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1 (a) Simplify:

(i)  $x^{\frac{3}{2}} \times x^{\frac{1}{2}}$ ; (1 mark)

(ii)  $x^{\frac{3}{2}} \div x$ ; (1 mark)

(iii)  $\left(x^{\frac{3}{2}}\right)^2$ . (1 mark)

(b) (i) Find  $\int 3x^{\frac{1}{2}} dx$ . (3 marks)

(ii) Hence find the value of  $\int_1^9 3x^{\frac{1}{2}} dx$ . (2 marks)

2 The  $n$ th term of a geometric sequence is  $u_n$ , where

$$u_n = 3 \times 4^n$$

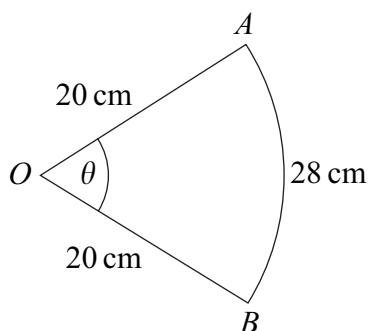
(a) Find the value of  $u_1$  and show that  $u_2 = 48$ . (2 marks)

(b) Write down the common ratio of the geometric sequence. (1 mark)

(c) (i) Show that the sum of the first 12 terms of the geometric sequence is  $4^k - 4$ , where  $k$  is an integer. (3 marks)

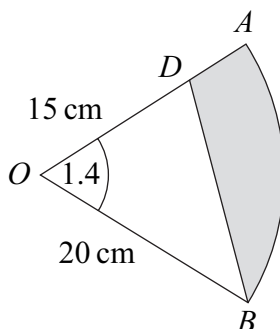
(ii) Hence find the value of  $\sum_{n=2}^{12} u_n$ . (1 mark)

- 3 The diagram shows a sector  $OAB$  of a circle with centre  $O$  and radius 20 cm. The angle between the radii  $OA$  and  $OB$  is  $\theta$  radians.



The length of the arc  $AB$  is 28 cm.

- (a) Show that  $\theta = 1.4$ . (2 marks)
- (b) Find the area of the sector  $OAB$ . (2 marks)
- (c) The point  $D$  lies on  $OA$ . The region bounded by the line  $BD$ , the line  $DA$  and the arc  $AB$  is shaded.



The length of  $OD$  is 15 cm.

- (i) Find the area of the shaded region, giving your answer to three significant figures. (3 marks)
- (ii) Use the cosine rule to calculate the length of  $BD$ , giving your answer to three significant figures. (3 marks)

Turn over ►

4 An arithmetic series has first term  $a$  and common difference  $d$ .

The sum of the first 29 terms is 1102.

(a) Show that  $a + 14d = 38$ . (3 marks)

(b) The sum of the second term and the seventh term is 13.

Find the value of  $a$  and the value of  $d$ . (4 marks)

5 A curve is defined for  $x > 0$  by the equation

$$y = \left(1 + \frac{2}{x}\right)^2$$

The point  $P$  lies on the curve where  $x = 2$ .

(a) Find the  $y$ -coordinate of  $P$ . (1 mark)

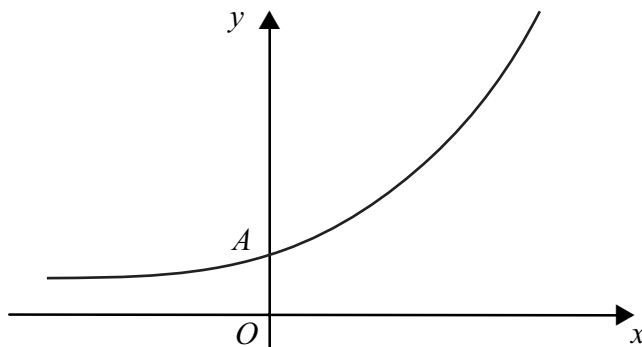
(b) Expand  $\left(1 + \frac{2}{x}\right)^2$ . (2 marks)

(c) Find  $\frac{dy}{dx}$ . (3 marks)

(d) Hence show that the gradient of the curve at  $P$  is  $-2$ . (2 marks)

(e) Find the equation of the normal to the curve at  $P$ , giving your answer in the form  $x + by + c = 0$ , where  $b$  and  $c$  are integers. (4 marks)

- 6 The diagram shows a sketch of the curve with equation  $y = 3(2^x + 1)$ .



The curve  $y = 3(2^x + 1)$  intersects the  $y$ -axis at the point  $A$ .

- (a) Find the  $y$ -coordinate of the point  $A$ . *(2 marks)*
- (b) Use the trapezium rule with four ordinates (three strips) to find an approximate value for  $\int_0^6 3(2^x + 1) dx$ . *(4 marks)*
- (c) The line  $y = 21$  intersects the curve  $y = 3(2^x + 1)$  at the point  $P$ .

- (i) Show that the  $x$ -coordinate of  $P$  satisfies the equation

$$2^x = 6 \quad (1 \text{ mark})$$

- (ii) Use logarithms to find the  $x$ -coordinate of  $P$ , giving your answer to three significant figures. *(3 marks)*

**Turn over for the next question**

**Turn over ►**

- 7 (a) Sketch the graph of  $y = \tan x$  for  $0^\circ \leq x \leq 360^\circ$ . (3 marks)
- (b) Write down the **two** solutions of the equation  $\tan x = \tan 61^\circ$  in the interval  $0^\circ \leq x \leq 360^\circ$ . (2 marks)
- (c) (i) Given that  $\sin \theta + \cos \theta = 0$ , show that  $\tan \theta = -1$ . (1 mark)
- (ii) Hence solve the equation  $\sin(x - 20^\circ) + \cos(x - 20^\circ) = 0$  in the interval  $0^\circ \leq x \leq 360^\circ$ . (4 marks)
- (d) Describe the single geometrical transformation that maps the graph of  $y = \tan x$  onto the graph of  $y = \tan(x - 20^\circ)$ . (2 marks)
- (e) The curve  $y = \tan x$  is stretched in the  $x$ -direction with scale factor  $\frac{1}{4}$  to give the curve with equation  $y = f(x)$ . Write down an expression for  $f(x)$ . (1 mark)

- 8 (a) It is given that  $n$  satisfies the equation

$$\log_a n = \log_a 3 + \log_a(2n - 1)$$

Find the value of  $n$ . (3 marks)

- (b) Given that  $\log_a x = 3$  and  $\log_a y - 3 \log_a 2 = 4$ :
- (i) express  $x$  in terms of  $a$ ; (1 mark)
- (ii) express  $xy$  in terms of  $a$ . (4 marks)

**END OF QUESTIONS**

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