General Certificate of Education
June 2006
Advanced Level Examination

MATHEMATICS **MM05**
Unit Mechanics 5

Friday 23 June 2006 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM05.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\ \text{m s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 Anne has made a simple pendulum which has a period of 4 seconds.

Show that the length of Anne's pendulum is $\frac{4g}{\pi^2}$. *(3 marks)*

2 A particle moves with simple harmonic motion on a straight line between the points P and Q. The amplitude of this motion is 0.3 m.

When the particle is 0.06 m from P, its speed is $0.9\,\mathrm{m\,s^{-1}}$.

(a) Show that the period of motion is $\frac{2\pi}{5}$ seconds. *(5 marks)*

(b) Find the magnitude of the maximum acceleration of the particle. *(2 marks)*

3 A particle moves so that, at time t, its polar coordinates (r, θ) with respect to a fixed origin, O, are such that $r = \frac{a}{1 + 5\cos\theta}$, where a is a constant.

(a) At the point A, the value of r is a minimum.

Show that at this point $r = \frac{a}{6}$. *(2 marks)*

(b) Show that $\dot{r} = \frac{5r^2}{a}\dot{\theta}\sin\theta$. *(3 marks)*

(c) Show that $\ddot{r} = \frac{5a\dot{\theta}^2}{36}$ when the particle is at A. *(6 marks)*

(d) The radial acceleration of the particle is $-\frac{\lambda}{r^2}$, where λ is a constant.

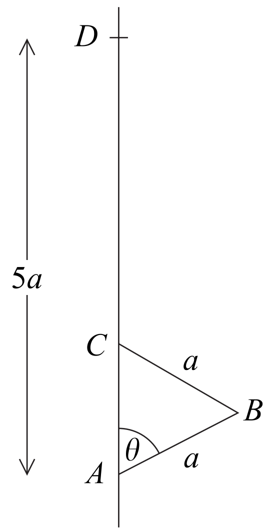
Find the speed of the particle at A in terms of λ and a. *(5 marks)*

4 A smooth rod passes through two points A and D, where D is a distance $5a$ vertically above A.

Two uniform rods, AB and BC, each of mass $2m$ and length a, are smoothly pivoted at B. The rod AB is smoothly pivoted at A, and the rod BC has the end C attached to a light smooth ring that can move freely on the vertical rod AD.

The ring, attached to the rod BC at C, is joined to the point D by a light spring, which has modulus of elasticity $6mg$, and natural length $3a$.

The rod AB is inclined at an angle θ to the vertical with the ring above A, as shown in the diagram.



(a) Show that V, the total potential energy of the system, is given by

$$V = 4mga(1 - \cos\theta + \cos^2\theta)$$

where the gravitational potential energy is taken to be zero at the level of A. *(6 marks)*

(b) Find the two values of θ in the range $0 \leqslant \theta \leqslant \frac{\pi}{2}$ for which the system is in equilibrium. *(4 marks)*

(c) Determine, for each of these values of θ, whether the system is in stable or unstable equilibrium. *(4 marks)*

Turn over for the next question

5 A car travels along a straight horizontal road. At time t, its speed is v and the total resistance to motion has magnitude kv, where k is a constant. The car is powered by a rocket, which ejects burnt fuel backwards at a constant rate λ and at a constant speed V relative to the car.

The initial mass of the car and the fuel is M.

(a) By considering linear momentum, show that the acceleration of the car along the road is

$$\frac{\lambda V - kv}{M - \lambda t}$$

(7 marks)

(b) The initial speed of the car is zero. Find its speed at time t. *(6 marks)*

(c) Initially, the mass of the fuel is 75% of the mass M. When $t = T_0$, all of the fuel has been burnt. Find T_0 in terms of M and λ. *(3 marks)*

6 A particle P, of mass $2m$, is suspended from a fixed point O by a light elastic string of natural length a and modulus of elasticity $8mn^2a$, where n is a positive constant. The particle is released from rest when $t = 0$ at a point A, where A is vertically below O and $OA = a$.

When the particle is moving with speed v, it experiences air resistance of magnitude $4mnv$.

(a) The displacement of P below A at time t is x. Show that x satisfies the equation

$$\frac{d^2x}{dt^2} + 2n\frac{dx}{dt} + 4n^2x = g$$

(3 marks)

(b) Find x in terms of n, g and t. *(12 marks)*

(c) Hence show that the first time the particle comes to rest is when $t = \dfrac{\pi}{\sqrt{3}n}$. *(4 marks)*

END OF QUESTIONS