

## MATHEMATICS II

Thursday 10th March 2005
Time allowed 2 hours
You may try the questions in any order and you are not expected to complete them all.

## Show all working.

1. A girl is making a rug. The rug is a sector of a circle of radius 2 m and angle $160^{\circ}$, as shown below

(a) Calculate the area of the rug.

The girl wishes to sew a frill along the arc AB .
(b) Calculate the length of the frill.

On measuring the frill the girl finds that she has only 5 m . If she decides to keep the radius as 2 m , calculate the angle of the sector which would allow her to use all of her frill.

2. It tıve lobsters and three crabs cost む146.53 and two lobsters and sev cost $£ 101.01$ calculate the cost of one lobster.
3. (a) Calculate $2 \%$ of $2 \frac{1}{2}$.
(b) Calculate $3 \%$ of $3 \frac{1}{3}$
(c) Calculate $4 \%$ of $4 \frac{1}{4}$.
(d) Calculate $5 \%$ of $5 \frac{1}{5}$
(e) Write down a general formula which summarises all of the above calculations.
(f) Justify your answer.
4.


The diagram shows a plot of land which is 11 m long by 9 m wide. At one end of the plot is a rectangular flower bed which is $x \mathrm{~m}$ wide, and in the middle is a rectangular fish pond, $(2 x) \mathrm{m}$ long by $(x) \mathrm{m}$ wide. The rest of the plot is set to lawn. The area of the lawn is $64 \mathrm{~m}^{2}$.
(a) Show that $2 x^{2}+9 x-35=0$
(b) Find the value of $x$.

5. I own a swimming pool. In order to fill it $I$ have a choice of four differe hoses. If I use hose A it will take 4 hours to fill. If I use hose B it will take hours. And if I use either hose C or hose D it will take 12 hours.

How long will it take to fill the pool if I use
(a) hoses C and D ?
(b) hoses B and C?
(c) hoses A and B and C?
6. This year the Radley College Scholarship Beetle is tackling a solid sphere of radius 12 cm . The beetle is at the lowest point, A, trying to get to the highest point, $P$.


The point C is on the surface of the sphere, 12 cm above the ground, and the point $B$ is on the surface of the sphere 6 cm above the ground. Find how far the beetle goes if it
(a) burrows in a straight line from A to P .
(b) burrows in a straight line from A to C then in a straight line from C to P .
(c) crawls over the surface in the shortest possible distance from A to P .
(d) crawls over the surface in the shortest possible distance from A to B, and then burrows in a straight line from B to P .


The diagram shows a rectangle ABCD in which $\mathrm{AB}=(x+8) \mathrm{cm}$ and $\mathrm{BC}=(x+1) \mathrm{cm}$. Find the value of $x$ in the following cases. In each case you should write down an equation, and solve it
(a) the perimeter of the rectangle is 20 cm
(b) the area of the rectangle is $44 \mathrm{~cm}^{2}$
(c) the diagonal, AC , is of length 13 cm .
8.


The diagram shows a tank in the shape of a cube of side 10 m . Water is poured into the tank to a depth of 3 m .

A cube of side 3 m is placed on the bottom of the tank.
(a) Calculate the amount by which the depth of the water rises.

The cube is removed and replaced with a cube of side 4 m .
(b) Calculate the amount by
$y$.


This year the Radley College Scholarship Tortoise, T, is tethered to the corner D of a square box of side 2 m . The box is placed in a large lawn beside a long wall. The side, AD , of the box is 2 m from the wall. The rope has length 9 m , and initially CDT $=45^{\circ}$.

The tortoise sets off in an anticlockwise direction, keeping the rope taut.
(a) Calculate how far he walks before bumping into the wall.

Assuming now that he can walk in any direction
(b) calculate the area of grass that he can reach.
10. The minute hand of a clock on a tower is 8 m long. At 3 o'clock a mouse is standing by the tip of the minute hand, and it sets off in a straight line towards the ' 6 ' moving at 1 cm per second. Show that the mouse will reach the ' 6 ' before the tip of the minute hand.

After reaching the ' 6 ' the mouse turns and heads off in a straight line towards the ' 9 'still moving at 1 cm per second. Will it get to the ' 9 ' before the tip of the minute hand?
11. A robot is walking across a courtyard paved with square paving slabs. l to walk from the South-West corner to North-East corner of the courtyard can only move North or East. It cannot move South, West or diagonally. It wants to know how many different ways there are of crossing the courtyard.

Example:


There are exactly 2 ways of getting across this 2 by 2 courtyard.
(a) How many different ways are there of getting across a 3 by 3 courtyard?


The robot wants to eat a nail that has been left on a 5 by 5 courtyard.
(b) How many different ways are there of crossing the courtyard by passin through to the square with the nail on it, in each of the following cases
i)

ii)

12. A pair of axes is drawn and a triangle ABC formed by joining the points A $(1,1), B(3,1)$ and $C(3,5)$.
(i) What is the area of the triangle?

The triangle $\mathrm{A}^{\prime} \mathrm{B}^{\prime} \mathrm{C}^{\prime}$ is now formed by multiplying the $y$ co-ordinates of $\mathrm{A}, \mathrm{B}$, and C by 3 (the $x$ co-ordinates are not changed).
(ii) What is the area of triangle $\mathrm{A}^{\prime} \mathrm{B}^{\prime} \mathrm{C}^{\prime}$ ?

The triangle $\mathrm{A} " \mathrm{~B} " \mathrm{C}$ " is now formed by multiplying the $x$ co-ordinates of $\mathrm{A}^{\prime}, \mathrm{B}^{\prime}$ and $\mathrm{C}^{\prime}$ by 6 (the $y$ co-ordinates are left unchanged).
(iii) What is the area of triangle $\mathrm{A}^{\prime \prime} \mathrm{B}^{\prime \prime} \mathrm{C}^{\prime \prime}$ ?
(iv) Suppose that, in parts (i), and (ii) above, the $x$ and $y$ co-ordinates of $\mathrm{A}, \mathrm{B}$ and C were in fact multiplied by positive whole numbers $h$ and $k$ respectively (rather than by 6 and 3 ). What would the area of $\mathrm{A} " \mathrm{~B} " \mathrm{C} "$ have been then?
(v) If the area of triangle $\mathrm{A}^{\prime \prime} \mathrm{B}^{\prime} \mathrm{C}^{\prime \prime}$ turned out to be $48 \mathrm{~cm}^{2}$, what are the possible values of $h$ and $k$ ? (remember that $h$ and $k$ are positive whole numbers).

