RADLEY COLLEGE
Entrance Scholarships

MATHEMATICS I

Thursday 13th February 2003
Time allowed 90 minutes

You may try the questions in any order and are not expected to complete them all.

Show all working.

1. (No calculating aids are to be used in this question)
a) Work out exactly

i) 60.8×2.71

ii) $172.02 \div 47$

b) Give the answers to the following as fractions in their simplest form

i) $\frac{13}{15} - \frac{7}{10}$

ii) $7\frac{1}{2} \div 5\frac{1}{4}$

iii) $\left(7\frac{2}{3} + 1\frac{4}{5}\right) \times 1\frac{5}{22}$

c) Give the answers to the following in standard form.

i) $\left(7 \times 10^{6}\right) + \left(5 \times 10^{6}\right)$

ii) $\left(6 \times 10^{3}\right) \times \left(4 \times 10^{-4}\right)$

2. (No calculating aids are to be used in this question)

Work out as simply as possible

a) $923^2 - 77^2$

b) $38^2 + (93 \times 38) - (38 \times 31)$

c) $(83 \times 47) + (17 \times 24) + (36 \times 83) - (41 \times 17)$

d) $\dfrac{456^2 + (456 \times 44)}{45.6 \times 25}$

3. a) Multiply out and simplify

i) $(2x + y)^2$

ii) $(3x + y)(9x^2 - 3xy + y^2)$

b) Factorise fully

i) $12x^2y + 16xy^2$

ii) $27 - 12x^2$

iii) $x^2 - 11x + 18$

c) Simplify

i) $\dfrac{xy + xz}{y^2 - z^2}$

ii) $\dfrac{x^2}{y^3} \div x^2y^2$

4. Solve each of these equations for x

a) $\frac{3x+1}{2} + \frac{2x+1}{7} = 6$

b) $2x^2 - 6x = 0$

c) $\frac{42}{x+2} + 6 = \frac{78}{x+2}$

d) $(3x+1)(x+2) - 3x^2 = 37$

5. Rearrange each of the following formulae to make x the subject

a) $a = bx - c$

b) $\frac{a}{x+b} = \frac{c}{x+d}$

c) $\sqrt{x-a} = b$

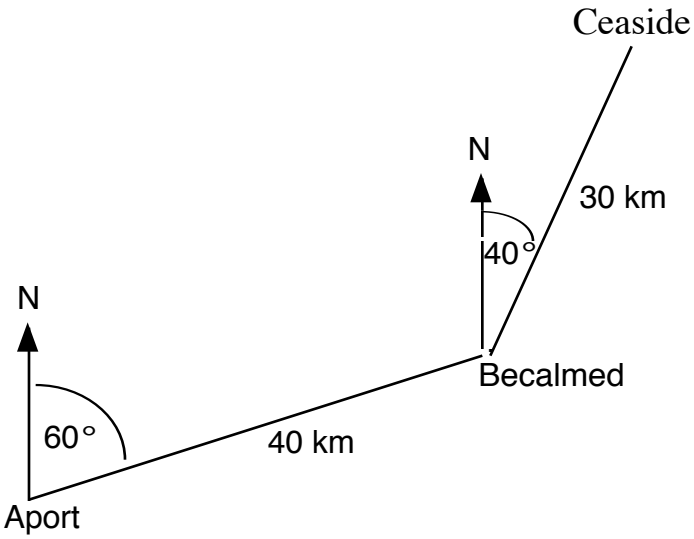
6. In cricket a batsman's average is calculated by dividing the total number of runs he has scored by the number of times he is out. So a batsman who has played 11 innings scoring a total of 400 runs and has been not out three times has an average of 50. $(400 \div (11-3) = 50)$.

Matthew Matics has become a keen cricketer. In the last match of the season, he scored 63 runs before he was out. He works out that this improved his season's average by 5. Letting x be the total he had scored before the final match and letting y be the number of times he had been out previously, write down an equation in x and y and show that it simplifies to $x + 5y^2 - 58y = 0$.

Then he realises that had he not been out (but still have scored 63 runs), his season's average would have been improved by 9 in the last match. Write down and simplify an equation using this piece of information.

Find the values of x and y, and thus find Matthew's batting average for the season.

7. A yachtsman sails 40 km on a bearing 060° from Aport to Becalmed, and then changes direction to sail 30 km on 040° to get to Ceaside.



a) How far East of Aport is Becalmed?
b) How far North of Aport is Becalmed?
c) How far East of Aport is Ceaside?
d) How far North of Aport is Ceaside?
e) Had the yachtsman wanted to sail directly from Aport to Ceaside, how far would it have been and on what bearing?

8. For any positive integer, n, we define $n!$ to be the product of all the integers between 1 and n inclusive.

So, for example, $5! = 1 \times 2 \times 3 \times 4 \times 5 = 120$

(a) Work out (i) $3!$
(ii) $6!$
(iii) $1!$

(b) For two positive integers, a and b, where $a > b$, we define an operation $*$ such that $a * b = \dfrac{a!}{(a-b)!}$

(i) Show that $6 * 2 = 30$

(ii) Work out $7 * 3$.

(c) Show that $n * 1 = n$, and work out a similar expression for $n * 2$.

(d) Solve the equation $n * 2 = 8n -$ [redacted]