RADLEY COLLEGE
Entrance Scholarships

MATHEMATICS I

Friday 23rd February 2001

Time allowed - $1\frac{1}{2}$ hours

You may try the questions in any order and are not expected to complete them all.

Show all working.

1 (No calculating aids are to be used in this question)

a) Work out exactly

i) 0.53×87.2

ii) $77.832 \div 9.2$

b) Give the answers to the following as fractions in their simplest form

i) $\frac{5}{18} + \frac{2}{9}$

ii) $2\frac{7}{10} \times 2\frac{1}{12}$

iii) $\left(4\frac{4}{5} - 2\frac{1}{4}\right) \div 4\frac{1}{4}$

c) Give the answers to the following in standard form.

i) $\left(4 \times 10^{8}\right) - \left(8 \times 10^{7}\right)$

ii) $\left(8 \times 10^{-3}\right) \times \left(3 \times 10^{-2}\right)$

iii) $\left(2.5 \times 10^{4}\right) \div \left(5 \times 10^{-2}\right)$

2. (No calculating aids are to be used in this question)

Work out as simply as possible

a) $652^2 - 348^2$

b) $(47 \times 86) + (61 \times 47) - 47^2$

c) $(74 \times 23) + (33 \times 26) + (51 \times 74) - (59 \times 26)$

d) $\dfrac{89.5^2 - (89.5 \times 59.5)}{8.95 \times 60}$

3. a) Multiply out and simplify

i) $(2x+y)(x-3y)$

ii) $(a+b)(a^2 - ab + b^2)$

b) Factorise fully

i) $25xy^2 - 15x^2y$

ii) $x^3 - xy^2$

iii) $x^2 + 7x - 8$

c) Simplify

i) $\dfrac{2xy - 2y^2}{x^2 - y^2}$

ii) $\dfrac{x^2}{y} \div xy$

4. Solve each of these equations for x

a) $\frac{x+3}{4}+\frac{4x-5}{3}=7$

b) $4x^2-20x=0$

c) $\frac{96}{x+3}-3=\frac{60}{x+3}$

d) $(x-5)^2-(x-3)(x-8)=13$

5. Rearrange each of the following formulae to make x the subject

a) $a+bx=c$

b) $\frac{x-a}{x-b}=c$

c) $\sqrt{x^2-a^2}=b$

6. Matthew Matics has amassed a large collection of conkers. Being orderly, he has designed a tray with the conkers in neat rows to make a rectangle.

There are 23 more conkers in each row than there are rows. If the number of rows is denoted by x, write down an expression for the total number of conkers that he has.

He decides a better arrangement would be to increase the number of rows and have fewer conkers in each row.

The new number of rows is two fewer than twice the original number of rows. The new number of conkers in each row is 14 fewer than originally. Write down a second expression for the total number of conkers.

By equating the two expressions for the total number of conkers and then simplifying, show that x satisfies the equation $x^2-7x-18=0$. Solve this to find x and deduce how many conkers Matthew has.

7 If the ends of the diameter of a circle are joined by straight lines to any point on the circumference of the circle, the angle between those two straight lines is 90°.

In order to support the cover for a circular well, four straight wires AB, BC, CD, and DA are attached to the top of the well, as in the diagram.

If AB is 78 cm long and BC is 104 cm long and points A and C are at opposite ends of the diameter, find

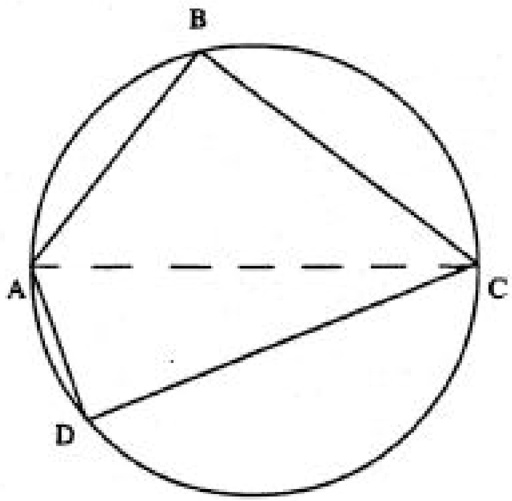
i) the angle BAC, and

ii) the length of the diameter AC.

If, furthermore, DA is 50 cm long, find

iii) the angle DAC, and

iv) the length CD.



8 The following is a method by which it is possible to find a better approximation to square root of a number, N,

"If x is an approximation, then $\frac{1}{2}\left(x+\frac{N}{x}\right)$ is a better approximation".

For example, to find better and better approximations to the square root of 2, starting from 1, the next approximation would be $\frac{1}{2}\left(1+\frac{2}{1}\right)=1.5$

and then this could be used in turn to find $\frac{1}{2}\left(1.5+\frac{2}{1.5}\right)=1.416666667$ to the accuracy of my calculator.

a) Repeat this process to find better and better approximations to $\sqrt{2}$, stopping when to the accuracy of your calculator no improvement is made.

b) Use this method to find $\sqrt{17}$ to the accuracy of your calculator using a sensible starting approximation.

c) A similar formula is $\frac{1}{3}\left(2x+\frac{N}{x^2}\right)$. Experiment with values of x and N to discover what this formula finds.