THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE II

Friday, June 15, 2001 — 1:15 to 4:15 p.m., only

Notice . . .

Scientific calculators must be available to all students taking this examination.

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

**DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.**

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of ≠ or in radical form.   [60]
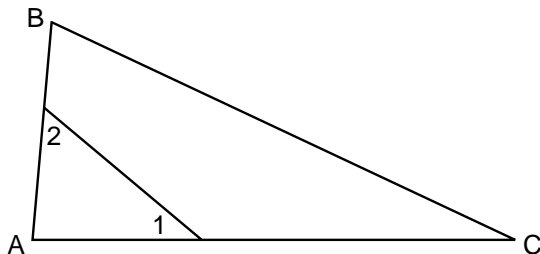
1 In the accompanying diagram of $\triangle DEF$, $\overline{AB} \parallel \overline{DE}$, $AF = 4$, $DF = 16$, and $FE = 20$. What is the length of $\overline{FB}$?
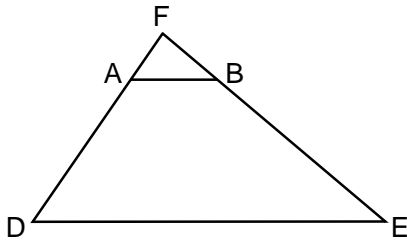
2 In $\triangle KID$, m$\angle K = 40$ and m$\angle D = 80$. Which side of $\triangle KID$ is the *shortest*?

3 The lengths of the sides of a triangle are 6, 10, and 14. Find the perimeter of the triangle formed by joining the midpoints of these sides.

4 In the accompanying diagram of parallelogram $ABCD$, m$\angle A = x + 5$ and m$\angle B = 2x + 1$. What is the value of x?
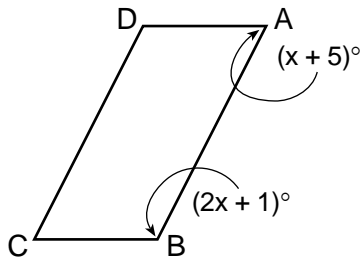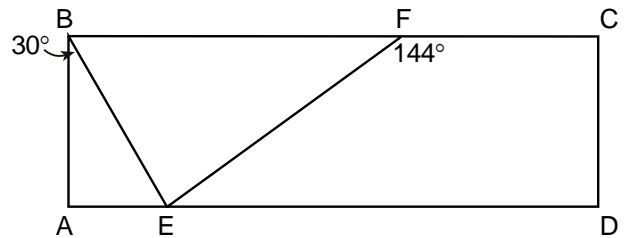
5 In the accompanying diagram of $\triangle ABC$, m$\angle 1 = 40$, m$\angle 2 = 55$, and m$\angle B = 70$. Find m$\angle C$.

6 A translation moves $A(-1,3)$ to $A'(-3,7)$. What are the coordinates of B', the image of $B(5,-3)$ under the same translation?

7 If $a \otimes b$ is a binary operation defined as $a \otimes b = \sqrt{2a^2 - b}$, express $3 \otimes 2$ in simplest form.

8 In the accompanying diagram of rectangle $ABCD$, m$\angle ABE = 30$ and m$\angle CFE = 144$. Find m$\angle BEF$.

9 Solve for x: $\dfrac{1}{4} + \dfrac{1}{x} = \dfrac{1}{3}$

10 Find, in radical form, the distance from point $(-2,3)$ to the origin.

11 What is the total number of different five-letter arrangements that can be formed using the five letters in the word "RADAR"?

12 Find the perimeter of a rhombus whose diagonals measure 12 and 16.

13 The coordinates of the vertices of $\triangle SUN$ are $S(-1,0)$, $U(5,0)$, and $N(3,3)$. Find the area of $\triangle SUN$.

14 A jar contains four red marbles, five white marbles, and seven blue marbles. If two marbles are drawn from the jar without replacement, what is the probability that both marbles will be blue?

15 How many degrees are in the sum of the exterior angles of a regular pentagon?

16 The altitude to the hypotenuse of a right triangle divides the hypotenuse into segments whose lengths are 12 and 50. What is the length of the altitude to the *nearest tenth*?

17 What is the inverse of *b* in the system below?

$*$	*a*	*b*	*c*	*d*
a	*d*	*c*	*a*	*b*
b	*c*	*d*	*b*	*a*
c	*a*	*b*	*c*	*d*
d	*b*	*a*	*d*	*c*

Directions (18–35): For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

18 In the accompanying diagram, *E* is the midpoint of $\overline{AB}$ and $\overline{CD}$.
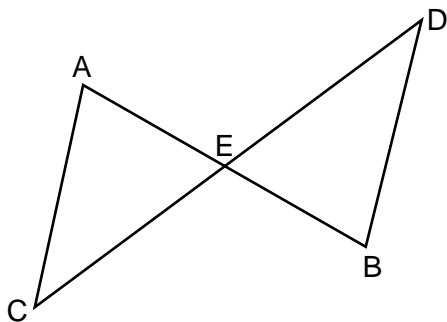

Triangle *AEC* can be proved congruent to triangle *BED* by

(1) AAS ≅ AAS   (3) SAS ≅ SAS
(2) ASA ≅ ASA   (4) SSS ≅ SSS

19 Which statement is the negation of "I drive and I do not speed"?

(1) I drive and I speed.
(2) I do not drive or I speed.
(3) I do not drive and I do not speed.
(4) I do not drive or I do not speed.

20 What is the image of point (–3,2) under a reflection in the origin?

(1) (–2,3)   (3) (–3,–2)
(2) (–2,–3)   (4) (3,–2)

21 Which equation represents the locus of points equidistant from points (2,3) and (2,9)?

(1) $y = 6$ (3) $x = 6$
(2) $y = -6$ (4) $x = -6$

22 What is the slope of the line segment that passes through points (1,3) and (5,13)?

(1) $\frac{1}{6}$ (3) $\frac{5}{2}$

(2) $\frac{2}{5}$ (4) 6

23 In the accompanying diagram of $\triangle AEB$, $\overline{EB}$ is extended to *R* and *K*, and m∠3 = m∠4 = 135.
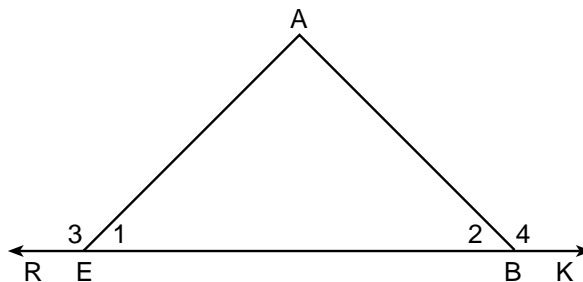
Triangle *AEB* must be

(1) equilateral
(2) acute and isosceles
(3) obtuse and isosceles
(4) right and isosceles

24 In the accompanying diagram of right triangle *RUN*, m∠*U* = 90, m∠*N* = 37, and *RN* = 21.
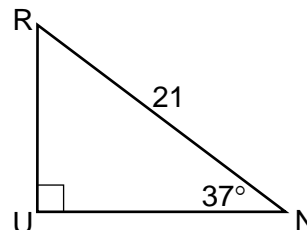
What is the length of $\overline{RU}$, expressed to the *nearest tenth?*

(1) 12.6   (3) 16.8
(2) 15.8   (4) 34.9

25 If $a \rightarrow b$ and $c \rightarrow \sim b$ are true statements, what is a valid conclusion?

(1) $a \rightarrow c$ (3) $\sim b \rightarrow a$
(2) $a \rightarrow \sim b$ (4) $a \rightarrow \sim c$

26 Which statement is the converse of "If a quadrilateral is a square, then the diagonals are perpendicular"?

(1) If the diagonals of a quadrilateral are not perpendicular, then it is not a square.
(2) If the diagonals of a quadrilateral are perpendicular, then it is a square.
(3) If a quadrilateral is not a square, then the diagonals are not perpendicular.
(4) If a quadrilateral is a square, then the diagonals are not perpendicular.

27 A committee of four is to be chosen from a group of two men and four women. From this group, how many different committees could consist of exactly one man and three women?

(1) $_4C_1 \cdot {_4}C_3$ (3) $_2C_1 \cdot {_4}C_3$
(2) $_6C_1 \cdot {_6}C_3$ (4) $_6C_4$

28 Which set of numbers can represent the lengths of the sides of a triangle?

(1) {2,3,5}   (3) {5,5,10}
(2) {4,8,13}   (4) {5,6,10}

29 What is the sum of $\frac{2x-5}{3}$ and $\frac{x+5}{2}$?

(1) $\frac{7x-5}{6}$ (3) $\frac{3x}{5}$
(2) $\frac{7x+5}{6}$ (4) $\frac{3x-5}{6}$

30 What are the roots of the equation $x^2 - 3x + 1 = 0$?

(1) $\frac{3 \pm \sqrt{5}}{2}$ (3) $\frac{3 \pm \sqrt{13}}{2}$
(2) $\frac{-3 \pm \sqrt{5}}{2}$ (4) $\frac{-3 \pm \sqrt{13}}{2}$

31 What is an equation of the axis of symmetry of the graph of the parabola $y = 2x^2 - 8x + 7$?

(1) $y = -2$ (3) $x = -2$
(2) $y = 2$ (4) $x = 2$

32 Which property is illustrated by the equation $\heartsuit + (\square + 0) = (\heartsuit + \square) + 0$?

(1) distributive
(2) associative property for addition
(3) commutative property for addition
(4) additive inverse

33 Which equation represents a circle with center $(1,-3)$ and radius 4?

(1) $(x-1)^2 + (y+3)^2 = 16$
(2) $(x-1)^2 + (y+3)^2 = 4$
(3) $(x+1)^2 + (y-3)^2 = 16$
(4) $(x+1)^2 + (y-3)^2 = 4$

34 In the accompanying diagram, $\triangle ABC$ is an equilateral triangle with a perimeter of 30.
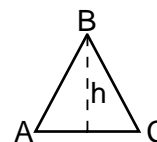
What is the length of altitude h of this triangle?

(1) $5\sqrt{2}$ (3) $10\sqrt{2}$
(2) $5\sqrt{3}$ (4) $10\sqrt{3}$

35 In the accompanying diagram of $\triangle ABC$, altitude $\overline{CD}$ is drawn to the hypotenuse, separating the triangle into two smaller triangles, ADC and BDC.
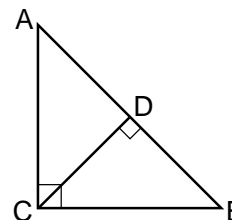
Which statement about these triangles is *not* always true?

(1) The corresponding angles of all three triangles are congruent.
(2) The corresponding sides of all three triangles are proportional.
(3) The two smaller triangles, ADC and BDC, are congruent.
(4) The two smaller triangles, ADC and BDC, are similar.

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer three questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [30]

36 Find the area of quadrilateral *QUAD,* whose vertices are *Q*(–5,5), *U*(4,2), *A*(1,–3), and *D*(–3,–4). [10]

37 *a* Solve the following system of equations algebraically:

$$y = x^2 - 3x - 10$$
$$3x - y = 19 \quad [6]$$

   b Simplify: $\dfrac{x^2 - 36}{5x - 30} \div \dfrac{5x + 30}{10}$ [4]

38 The vertices of △*PEN* are *P*(1,2), *E*(3,0), and *N*(6,4).

   a On graph paper, draw and label △*PEN*. [1]

   b Graph and state the coordinates of △*P'E'N',* the image of △*PEN* after a reflection in the *y*-axis. [3]

   c Graph and state the coordinates of △*P"E"N",* the image of △*PEN* under the translation $(x,y) \rightarrow (x + 4, y - 3)$. [3]

   d Graph and state the coordinates of △*P"'E"'N"'*, the image of △*PEN* after a dilation of 2. [3]

39 The vertices of △*PQR* are *P*(8,6), *Q*(–1,13), and *R*(5,–5).

   a Find the coordinates of *M*, the midpoint of $\overline{QR}$. [2]

   b Prove that $\overline{PM}$ is perpendicular to $\overline{QR}$. [4]

   c Prove that △*PQR* is isosceles. [4]

40 *a* In right triangle *DEF*, m∠*D* = 28, m∠*E* = 90, and *DE* = 500.

   Find:
   (1) *EF,* to the *nearest tenth* [2]
   (2) the perimeter of △*DEF*, to the *nearest integer* [4]

   b Solve for *x*: $\dfrac{5}{x} + \dfrac{2x}{x + 3} = 2$ [4]

GO RIGHT ON TO THE NEXT PAGE. ⇨

Answers to the following questions are to be written on paper provided by the school.

Part III

Answer one question from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [10]

41 Given: If Lorraine does not tour Europe and Elizabeth goes on a cruise, then Barbara does not go to tennis camp.

If Elizabeth does not go on a cruise, then MaryAnn goes to the beach.

If MaryAnn goes to the beach, then Robert goes to summer school.

Barbara goes to tennis camp.

Lorraine does not tour Europe.

Let *L* represent: "Lorraine tours Europe."
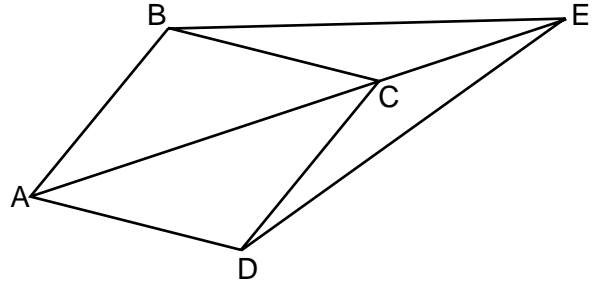Let *E* represent: "Elizabeth goes on a cruise."
Let *B* represent: "Barbara goes to tennis camp."
Let *M* represent: "MaryAnn goes to the beach."
Let *R* represent: "Robert goes to summer school."

Prove: Robert goes to summer school. [10]

42 Given: rhombus *ABCD*, diagonal $\overline{AC}$ extended through *C* to *E*, $\overline{BE}$, and $\overline{DE}$.

Prove: $\overline{BE} \cong \overline{DE}$ [10]

SEQUENTIAL MATH – COURSE II

Friday, June 15, 2001 — 1:15 to 4:15 p.m., only

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Part I Score	. . . .
Part II Score	. . . .
Part III Score	. . . .
Total Score	. . . .
Rater's Initials:	. . . .

ANSWER SHEET

Pupil . . . .   Sex: ☐ Male ☐ Female   Grade . . . .

Teacher . . . . School . . . .

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer 30 questions from this part.

1	11	21	31 . . . .
2	12	22	32 . . . .
3	13	23	33 . . . .
4	14	24	34 . . . .
5	15	25	35 . . . .
6	16	26	
7	17	27	
8	18	28	
9	19	29	
10	20	30	

Your answers for Part II and Part III should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination, and that I have neither given nor received assistance in answering any of the questions during the examination.

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Signature

[8]