

0048.1S

WISKUNDE HG  
MATHEMATICS HG

(VRAESTEL 1)  
(PAPER 1)

MAART/MARCH 2006

PUNTE: 200  
MARKS: 200

3 URE  
3 HOURS



education

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Department:  
Education  
REPUBLIC OF SOUTH AFRICA

**SENIORSERTIFIKAAT-EKSAMEN – 2006**  
**SENIOR CERTIFICATE EXAMINATION - 2006**

**Hierdie vraestel bestaan uit 9 bladsye, 1 grafiekpapier en 1 inligtingsblad.**  
**This question paper consists of 9 pages, 1 graph paper and 1 information sheet.**

*Kopiereg voorbehou*

*Blaai om asseblief*

**INSTRUCTIONS**

Read the following instructions carefully before answering the questions:

1. This paper consists of **8** questions. Answer **ALL** the questions.
2. Clearly show **ALL** calculations, diagrams, graphs, et cetera you have used in determining the answers.
3. An approved calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
4. If necessary, answers should be rounded off to **TWO** decimal places, unless stated otherwise.
5. The attached graph paper must be used only for **QUESTION 8**. Detach it from your question paper, fill in your examination number and centre number and insert it in the **FRONT** of the answer book.
6. Number the answers **EXACTLY** as the questions are numbered.
7. Diagrams are not necessarily drawn to scale.
8. It is in your own interest to write legibly and to present the work neatly.
9. **An information sheet with formulae is included at the end of the question paper.**

**QUESTION 1**1.1 Solve for  $x$ :

1.1.1  $-3x^2 + 5x + 2 = 0$  (2)

1.1.2  $|x + 3| < 9$  (3)

1.1.3  $x - 7 - \sqrt{x - 5} = 0$  (6)

1.1.4  $4(2^x) + 3 = \frac{1}{2^x}$  (5)

1.2 Given:  $(x - 2)(x - k) = -4$ .1.2.1 For which values of  $k$  will the equation have real roots? (7)1.2.2 Find a value of  $k$  for which the roots are rational and unequal. (3)

1.3 Trevor employs a certain number of workers and pays each worker the same wage. His current daily wage bill is R5 880. A labour dispute has resulted in his workers demanding a wage increase of R10 per day. Trevor claims that he cannot afford this. He claims that only if he retrenches 4 workers will he be able to give them the increase that they demand. His daily wage-bill would then be R5 850.



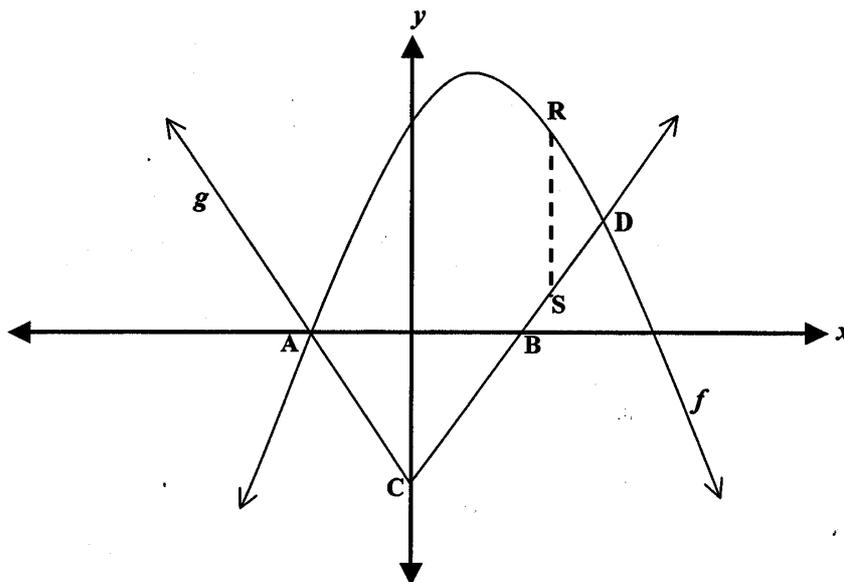
1.3.1 Calculate how many workers Trevor employs. (7)

1.3.2 How much does each worker earn per day, presently? (2)

**[35]**

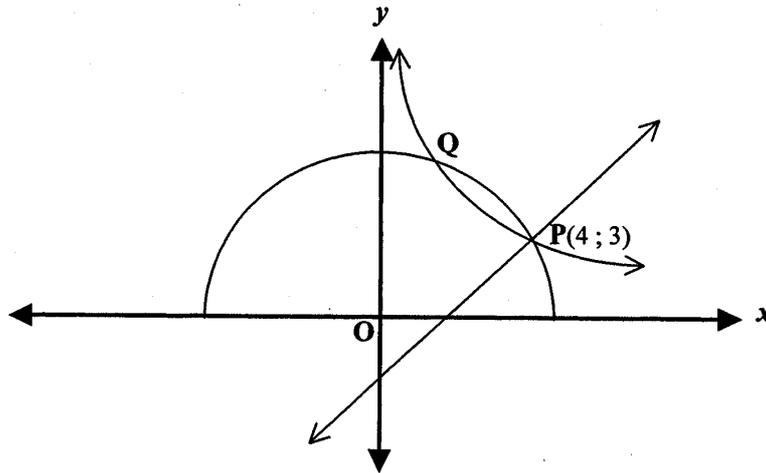
**QUESTION 2**

- 2.1 The sketch below shows the graphs of the parabola defined by  $f(x) = -x^2 + bx + c$  and the absolute value function defined by  $g(x) = |x| - 3$ .  
 The points A, B and C are the  $x$ - and  $y$ -intercepts of the graph of  $g$ . A and D are on both graphs.



- 2.1.1 Write down the co-ordinates of A. (2)
- 2.1.2 Given that the equation of the axis of symmetry of  $f$  is  $x = 1$ , show that the equation of the parabola is  $y = -x^2 + 2x + 15$ . (5)
- 2.1.3 It is further given that R and S are variable points on  $f$  and  $g$ , and that the straight line RS is parallel to the  $y$ -axis.
- (a) If S moves between C and D, write down an expression for the length of RS in terms of  $x$ . (3)
- (b) Determine the coordinates of R if RS is as large as possible. (5)

- 2.2 The sketch represents graphs of  $xy = k$  ( $x > 0$ ),  $x^2 + y^2 = r^2$  ( $y \geq 0$ ) and  $y = mx + c$ . All three graphs intersect at  $P(4; 3)$ . The straight line has the same gradient as the axis of symmetry of the hyperbola.



- 2.2.1 Determine the values of  $k$ ,  $r$ ,  $m$  and  $c$ . (6)
- 2.2.2 Write down the co-ordinates of  $Q$ . (2)
- 2.3  $S\left(\frac{1}{2}; \frac{1}{2}\right)$  is a point on the graph of  $f$  defined by  $f(x) = a^x$  ( $a > 0$ ).
- 2.3.1 Prove that  $a = \frac{1}{4}$ . (2)
- 2.3.2 Determine  $f^{-1}$  in the form  $f^{-1}(x) = \dots$  (2)
- 2.3.3 Calculate the value of  $x$  if  $f^{-1}(x) = -1,5$ . (3)
- 2.3.4 Sketch the graph of  $f$  and clearly indicate the co-ordinates of the intercepts with any of the axes. (2)

[32]

**QUESTION 3**

3.1 If  $(x + 2)$  is a common factor of  $f(x) = x^3 + ax^2 + 2b$  and  $g(x) = x^3 + ax - 4b$ , determine the values of  $a$  and  $b$ . (5)

3.2 A polynomial  $f(x)$  can be written in the form  $f(x) = (x + k).q(x) - 12$ . Calculate the value of  $k$  if  $(x - 3)$  is a factor of  $f(x)$  and  $q(x)$  leaves a remainder of 3 when divided by  $(x - 3)$ . (4)

[9]

**QUESTION 4**

4.1 Simplify to a single number **without using a calculator**:

4.1.1  $\sqrt[3]{(\sqrt{13} - \sqrt{5})^6} \cdot \sqrt[3]{(\sqrt{13} + \sqrt{5})^6}$  (4)

4.1.2  $3 \log \sqrt[3]{40} - 2 \log \frac{1}{5}$  (4)

4.2 Solve for  $x$ :

4.2.1  $3^{x+1} - 3^{x-1} = 24\sqrt{3}$  (5)

4.2.2  $7^x = 126(5^x)$  (round off to two decimal places) (3)

4.3 4.3.1 Prove that  $\log_{\frac{1}{a}} x = -\log_a x$ , for any  $a > 0$  (3)

4.3.2 Solve for  $x$ :  $\log_{10}(2x - 5) \leq \log_{\frac{1}{10}}(x - 3)$  (9)

[28]

**QUESTION 5**

5.1 The sum of the first 50 terms of an arithmetic series is 1 275. Calculate the sum of the 25<sup>th</sup> and 26<sup>th</sup> terms of this series. (6)

5.2 The sum of the first  $n$  terms of an arithmetic series is:  $S_n = \frac{3n^2 - n}{2}$ .

5.2.1 Determine  $S_{10}$ . (2)

5.2.2 Calculate the value of  $\sum_{r=5}^{10} T_r$ , where  $T_r$  is the  $r^{\text{th}}$  term of the series. (3)

- 5.3 The first term of a geometric sequence is 3 and the sum of the first 4 terms is 5 times the sum of the first 2 terms. The common ratio is greater than 1.

Calculate:

5.3.1 The first three terms of the sequence (7)

5.3.2 The value of  $n$  for which the sum to  $n$  terms will be 765 (4)

- 5.4 The first two terms of a convergent geometric series are  $m$  ( $m \neq 0$ ) and 6, in that order. The sum of the infinite series is 25. Calculate the values of  $m$ . (Check that these values are acceptable.)

(7)

[29]

### QUESTION 6

6.1 Determine  $\lim_{h \rightarrow 4} \frac{h^2 - h - 12}{16 - h^2}$  (3)

6.2 Given:  $f(x) = -\frac{x^2}{2} + x$

6.2.1 Determine  $f'(x)$ , using the definition of the derivative. (6)

6.2.2 Use your answer in QUESTION 6.2.1 to determine the value of  $\lim_{h \rightarrow 0} \frac{f(1+h) - f(h)}{h}$  (2)

6.2.3 A tangent to the graph of  $f$  has gradient  $-5$ , and  $x$ -intercept  $(a; 0)$ . Determine  $a$ . (6)

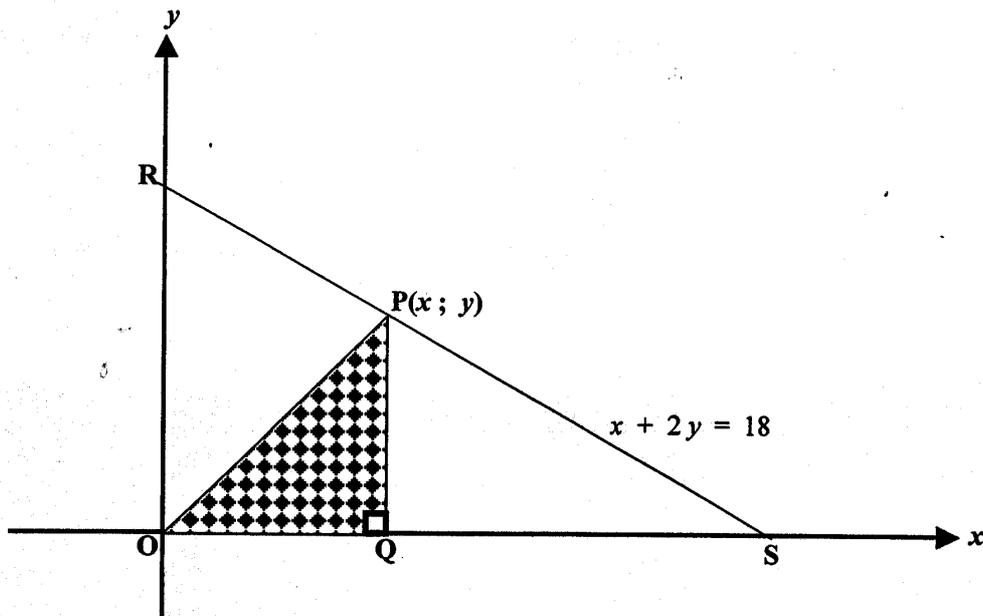
6.3 Determine  $\frac{dy}{dx}$  if  $y = \frac{5x^5 - 6x^{\frac{3}{2}} + 5}{x}$  (5)

[22]

**QUESTION 7**

- 7.1 Given:  $f(x) = -x^3 + 3x^2 - 4$
- 7.1.1 Determine the  $x$ - and  $y$ -intercepts of the graph of  $f$ . (7)
  - 7.1.2 Determine the coordinates of the turning points of  $f$ . (5)
  - 7.1.3 Sketch the graph of  $f$ . Show clearly all the turning points as well as the intercepts on the axes. (4)
  - 7.1.4 For which values of  $x$  is  $f$  increasing? (2)
  - 7.1.5 What is the maximum value of  $-x^3 + 3x^2 - 4$  if  $0 \leq x \leq 3$ ? (1)
  - 7.1.6 How many solutions does the equation  $f(x) = -5$  have? (1)

7.2 A point  $P$  lies on the line segment as shown.



If the equation of  $RS$  is given by  $x + 2y = 18$ ,  $0 \leq x \leq 18$ , and  $A$  is the area of the right-angled triangle  $OPQ$ , determine the coordinates of  $P$  so that the area of  $\Delta OPQ$  is as large as possible.

(8)

[28]

**QUESTION 8**

The owner of a pleasure boat is prepared to take a school group consisting of learners and adults on a cruise, provided that the group consists of not more than 60 people. In addition:

- (i) There must be at least 35 people in the group
- (ii) There must be at least 6 adults in the group
- (iii) There must be not more than 14 adults

Let  $x$  be the number of learners, and  $y$  the number of adults.

- 8.1 Give all the constraints in terms of  $x$  and  $y$ . (5)
- 8.2 If the group has 25 learners, what is the minimum number of adults that must accompany them? (1)
- 8.3 Eight adults offer to go on the cruise. What is the maximum number of learners that can be accommodated on the boat? (1)
- 8.4 If  $T$  is the amount in rand paid by the whole group, what is the cost per learner if  $T = 30x + 50y$ ? (2)
- 8.5 Now represent the constraints graphically on the graph paper provided and indicate the feasible region clearly. (5)
- 8.6 What is the composition of the group if the owner's income is as large as possible? (3)

**[17]****TOTAL: 200**

**Mathematics Formula Sheet (HG and SG)**  
**Wiskunde Formuleblad (HG en SG)**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$T_n = a + (n - 1)d \quad S_n = \frac{n}{2}(a + T_n) \quad \text{or / of } S_n = \frac{n}{2}(a + l)$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$T_n = ar^{n-1} \quad S_n = \frac{a(1 - r^n)}{1 - r} \quad (r \neq 1) \quad S_n = \frac{a(r^n - 1)}{r - 1} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1 - r} \quad (|r| < 1)$$

$$A = P \left( 1 + \frac{r}{100} \right)^n \quad \text{or / of} \quad A = P \left( 1 - \frac{r}{100} \right)^n$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x_3; y_3) = \left( \frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2} \right)$$

$$x^2 + y^2 = r^2$$

$$(x - p)^2 + (y - q)^2 = r^2$$

$$\text{In } \Delta ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \Delta ABC = \frac{1}{2} ab \cdot \sin C$$

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