SENIOR CERTIFICATE EXAMINATION

GAUTENG PROVINCIAL GOVERNMENT

FEBRUARY / MARCH

2007

ADDITIONAL MATHEMATICS

HG

302-1/0 E

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302 1 0E HG

X05

16 pages

GAUTENG DEPARTMENT OF EDUCATION

SENIOR CERTIFICATE EXAMINATION

ADDITIONAL MATHEMATICS HG

TIME: 3 hours

MARKS: 400

INSTRUCTIONS:

- This examination paper consists of FIVE sections.
- Section A is COMPULSORY.
- A further TWO sections must be answered from Sections B, C, D and E.
- Each section must **be answered in a separate answer book and the relevant section must be clearly indicated on the cover**. Place all answer books inside the answer book for Section A before handing them in.
- Unless otherwise indicated, non-programmable calculators may be used.
- The examination paper consists of 16 pages. Statistical tables and formula sheets can be found on pages, 14, 15 and 16 respectively.
- All essential calculations must be clearly shown.
- All angles are in radians and answers must also be given in radians.
- Writing must be legible.

SECTION A
COMPULSORY
CALCULUS

QUESTION 1

1.1 Determine the circumference and area of a sector of a circle with radius 2 and a central angle equal to $\frac{\pi}{6}$. Give the answer in terms of π. (6)

1.2 $f: x \rightarrow \arccos(x+1)$

1.2.1 Determine the domain of f. (4)

1.2.2 Sketch the graph of f. (6)

1.3 Answer this question without the use of a calculator: Determine:

1.3.1 $\arcsin\left(\sin \frac{11\pi}{6}\right)$ (6)

1.3.2 $\tan \frac{7\pi}{12}$ (6)

[28]

QUESTION 2

$$f(x) = \begin{cases} 2^x & \text{if} \quad x < 2 \\ 5 & \text{if} \quad x = 2 \\ -x+6 & \text{if} \quad 2 < x \leq 4 \\ 2 & \text{if} \quad x > 4 \end{cases}$$

2.1 Determine if f is continuous at the following points and substantiate fully. If not continuous, state what type of discontinuity it is:

2.1.1 The point where $x = 2$ (8)

2.1.2 The point where $x = 4$ (6)

2.2 Assume the function is continuous at $x = 4$. Determine if f is differentiable at $x = 4$, using algebraic methods. It is not required to work from first principles. (6)

[20]

QUESTION 3

Calculate, if possible:

3.1 $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\cos 2x}{\cos x - \sin x}$ (8)

3.2 $\lim\limits_{x\to\infty}\dfrac{2-x}{\sqrt{9x^2+1}}$ (6)

[14]

QUESTION 4

Determine the following derivatives:

4.1 $f'(x)$ from first principles if $f(x)=\sqrt{2x-1}$ (10)

4.2 $\dfrac{dy}{dx}$ if $y=\dfrac{\operatorname{cosec} 3x}{x^3}$ (8)

4.3 $D_x\left(\sin^3(\arctan x)\right)$ (8)

4.4 The n^{th} derivative of $f(x)=\dfrac{2}{1+2x}$ (12)

[38]

QUESTION 5

Given that $f(x) = x^2 - 4x$, determine the area under the graph of $f(x)$ and above the **X**-axis between $x = 3$ and $x = 6$. Use the technique of approximating the area by finding n strips of equal width and determining the Riemann sum for the area of these rectangles. Then let $n \to \infty$ **[20]**

QUESTION 6

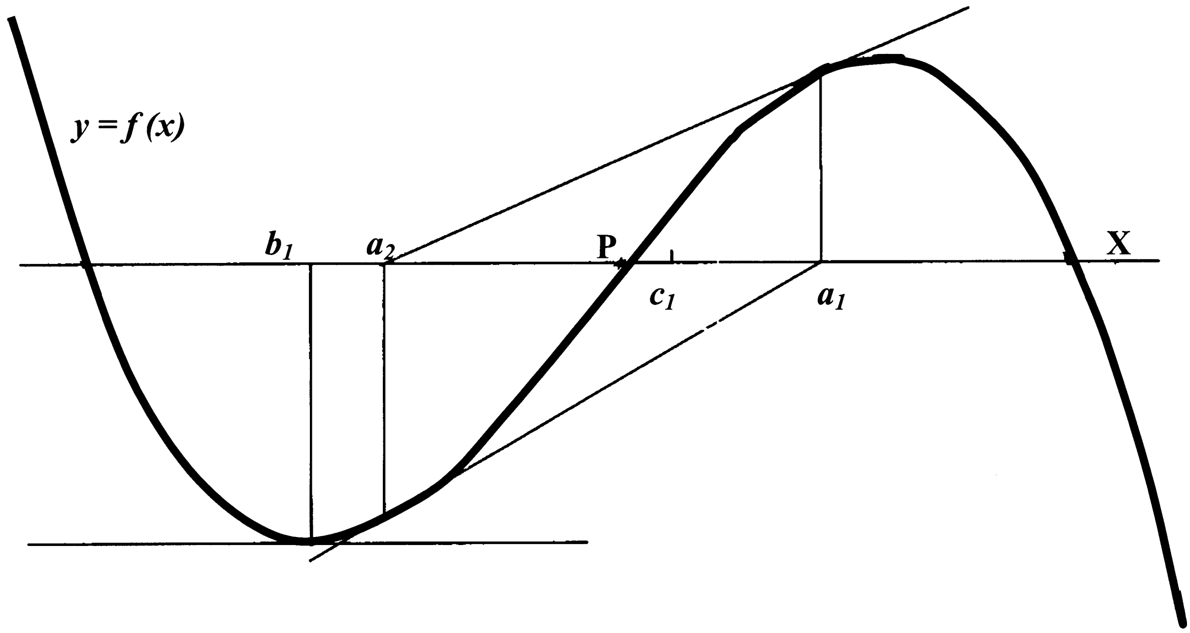
Determine the following integrals:

6.1 $\displaystyle\int\frac{\arctan 2x}{1+4x^2}\,dx$ (6)

6.2 $\displaystyle\int_0^{2/\sqrt{3}}\frac{dx}{9x^2+4}$ (12)

[18]

QUESTION 7



The curve $y = f(x)$ above has a zero point at **P**. Three Additional Mathematics candidates, Snap, Crackle and Pop are asked to find ***P*** using Newton's method. Each uses a different value as a first estimate.

7.1 Snap begins with a_1 but encounters a problem. Explain his problem from a graphical perspective. (6)

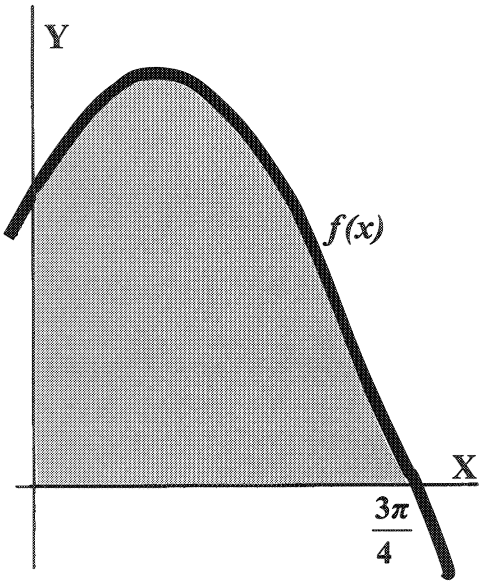
7.2 Crackle begins with b_1 and also encounters a problem. Explain his problem graphically. (4)

7.3 Pop begins with c_1 and finds the correct value of ***P***. Calculate this answer, correct to three decimal places, if $f(x) = -2x^3 + 2x^2 + 2x - 1$ and $c_1 = 0,5$. (10)

[20]

QUESTION 8

The function $f(x) = \sin x + \cos x$ is sketched below for the interval $\left[0; \frac{3\pi}{4}\right]$.



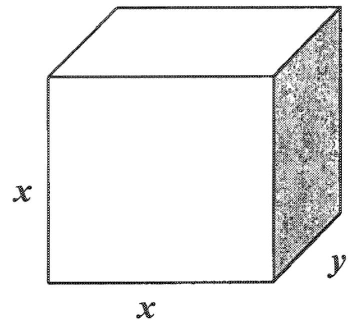
8.1 Calculate the area between $f(x)$ and the x axis between $x = 0$ and $x = \frac{3\pi}{4}$. (10)

8.2 Determine the volume of the solid of revolution obtained by rotating the curve, $f(x)$ between $x = 0$ and $x = \frac{3\pi}{4}$ about the x axis. (16)

[26]

QUESTION 9

Postal regulations dictate that a rectangular parcel may not exceed a certain size, namely that $x + 4y = 78$ (It may also be less)



9.1 Show that the formula, in terms of x, for the volume of such a parcel is given by $V(x) = 78x^2 - 4x^3$ (4)

9.2 Calculate the dimensions of the largest possible rectangular parcel with a square cross-section. Show that your answer will indeed give a maximum volume. (12)

[16]

TOTAL FOR SECTION A: [200]

Answer any **TWO** of the following **FOUR** sections.

SECTION B
FINANCIAL MATHEMATICS

Where appropriate, give all answers in this section to the nearest cent.

QUESTION 10

The annual cost and revenue functions for a wine farmer are given by

$C(q) = \frac{2}{3}q^3 + 10q^2 + 15\,000$ and

$R(q) = 40q^2 + 3\,600q$ where q is the number of hectares of vineyard under cultivation.

Calculate the maximum profit a farmer can make annually from wine farming. **[14]**

QUESTION 11

Mr Mizer takes out a loan from a bank which charges an interest rate of 12% per annum compound interest for 10 years and 13% per annum compound interest for the next 10 years. Write down a formula in terms of x which Mr Mizer can use to calculate the size of the loan, L, he can take out if he pays back x rand per year at the end of each year for 20 years. His first payment is one year after the loan is granted. It is not necessary to simplify the formula. **[12]**

QUESTION 12

Vuyo takes out a loan of R80 000 from a bank which charges 12% p.a., compounded quarterly for 2 years and thereafter 12% p.a. compounded monthly. He plans to pay back as follows:

- x rand after 1 year
- R10 000 1½ years after the loan is granted
- $2x$ rand 3 years after the loan is granted
- R30 000 4 years after the loan is granted, with which he wants to amortise the loan.

Calculate x **[14]**

QUESTION 13

Kloofwaters School has just purchased a new bus for R190 734,86. In 6 years' time the plan is to buy a replacement. By that time the one just purchased will be expected to have a trade-in value of R50 000. The inflation rate on the cost of new buses is 7% per annum.

13.1 What is the annual depreciation rate on a reducing balance which was used to calculate the trade-in value? (4)

13.2 What is the expected cost of a new bus in 6 years' time? (4)

13.3 A sinking fund is set up to provide for an annual service and to replace the bus in 6 years' time. The account used offers 12% per annum, compounded monthly.

13.3.1 Calculate the effective interest rate compounded yearly. Give the answer correct, as a percentage, to two decimal digits. (6)

13.3.2 If the annual service costs R4 000 and takes place at the end of each of the first 5 years, how much will have been spent for service costs in total at the end of 6 years? Use $i = 0{,}1268$. (10)

13.3.3 The sinking fund must be R265 266,60 in 6 years' time. This amount covers the service costs as well as the amount needed to purchase a new bus. Calculate the monthly payment into the sinking fund. Payments are made at the end of each month and end with the purchase of the bus. (10)

[34]

QUESTION 14

A fixed monthly payment of R1 000,00 is made into an account earning 6% interest per annum compounded monthly. This spans a period of 10 years.

14.1 Determine the value of the annuity at the end of the 10 years. Payments are made at the end of each month, starting at the end of month 5 and ending 4 months before the end of the 10 year period. (14)

14.2 After 10 years, the annuity of R152 661,73 now earns interest at 9% p.a. compounded monthly. No more payments are made. One month later deductions of R3 000 start. Calculate how many of these withdrawals can be made. (12)
[26]

TOTAL FOR SECTION B: [100]

SECTION C
ANALYTICAL GEOMETRY

QUESTION 15

Write all answers to this question correct to two decimal places.

15.1 Determine the obtuse angle between the lines $x + y - 4 = 0$ and $x + 7y - 7 = 0$. (8)

15.2 Determine the equation of the bisector of the acute angle between the lines in Question 15.1. (8)
[16]

QUESTION 16

16.1 Determine the equation of the common chord of the circles $x^2 + y^2 - 8x - 4y = 0$ and $x^2 + y^2 - 10x + 20 = 0$. (4)

16.2 Show that this is in fact a common tangent, i.e. that the circles touch at one point only. (8)
[12]

QUESTION 17

A parabola is defined by the equation $y^2 + 8x - 6y + 1 = 0$.

17.1 Write the equation in the form $(y - h)^2 = 4a(x - k)$. (8)

17.2 Hence find

17.2.1 the vertex.

17.2.2 the focus.

17.2.3 the directrix.

17.2.4 the length of the latus rectum perpendicular on the symmetrical axis. (8)

[16]

QUESTION 18

18.1 Show that the curve with the equation $9x^2 - 16y^2 + 54x - 32y = 79$ is a hyperbola by proving that its equation can also be written as $\frac{(x+3)^2}{16} - \frac{(y+1)^2}{9} = 1$. (10)

18.2 Write down the asymptotes of the curve. (8)

18.3 Hence sketch the graph of the hyperbola. (10)

[28]

QUESTION 19

The points $A(1 ; 0 ; 2)$ $B(2 ; -3 ; 0)$ $C(0 ; -2 ; 3)$ form the vertices of a triangle.

19.1 Determine the angle between the lines AC and BC. (Give your answer correct to one decimal place.) (12)

19.2 Hence determine the area of ΔABC. (Give your answer correct to two decimal places.) (4)

19.3 Determine the equation of the flat plane which contains ΔABC. (12)

[28]

TOTAL FOR SECTION C: [100]

SECTION D
ALGEBRA

QUESTION 20

20.1 Use mathematical induction to prove that $\sum_{k=1}^{n} \frac{1}{2^k} = 1 - \frac{1}{2^n}$

for all $n \in N$. (16)

20.2 Decompose $\frac{4}{(x^2+2)(x^2+1)}$ into partial fractions. (16)

20.3 $f(x) = x^3 - 3x^2 + 2x - 1$ with zeros **a**, **b** and **c**. Determine the value of $a^2 + b^2 + c^2$. (14)
[46]

QUESTION 21

21.1 State **Eisenstein's criterion**. (8)

21.2 Use Eisenstein's criterion to determine if the polynomial
$x^4 + 6x^3 + 18x^2 + 12$ is irreducible in **Z**[x]. (8)

21.3 If $1-\sqrt{2}$ is a zero of $f(x) = x^5 - 3x^4 - 4x^3 + 16x^2 - 5x - 5$, factorise $f(x)$ fully over **R**. (18)
[34]

QUESTION 22

$$f(x) = \frac{x^2 - 4x - 5}{2x - 1}$$

The graph of this function has no turning points and is increasing.

22.1 Determine the graph's intercepts with the axes. (6)

22.2 Determine all asymptotes of this function. Show all your calculations. (6)

22.3 Sketch the graph of $y = f(x)$, and show all asymptotes and intercepts with the axes. (8)
[20]

TOTAL FOR SECTION D: [100]

SECTION E
STATISTICS

Answers must be given correct to four decimal digits if necessary.

QUESTION 23

23.1 In how many ways can

23.1.1 3 prizes be given to 18 persons if all the prizes are the same and a person can win more than one prize? (4)

23.1.2 8 prizes be given to 18 persons if there is one first-, one second- and six third prizes? A person can only win one prize. (6)

23.2 How many matches must be scheduled for the future "Super 14" rugby, if each team must play twice against every other team? (6)

[16]

QUESTION 24

The following is known about events A and B:

- They are independent.
- P(A) = 2P(B).
- The probability that both of the two events will take place, is 0,625.

Determine P(A). **[16]**

QUESTION 25

"Super Sweets" is a company which manufactures sweets. At the end of each day all the lollipops that are rejected are thrown into a large container for the workers. Martin walks past this container of sweets 6 times and takes one lollipop each time without looking to see what colour it is.

Determine the probability of each event below as a percentage, if it is known that 15% of the lollipops in the container are green:

25.1 Martin takes out two green lollipops. (8)

25.2 Martin takes out two or more green lollipops. (8)

[16]

QUESTION 26

A probability density function is defined as follows:

$$f(x) = \begin{cases} \boldsymbol{a(6-x); 0 \le x \le 6} \\ \textbf{0 elsewhere} \end{cases} \quad \text{for some constant } \boldsymbol{a}$$

26.1 Determine the value of ***a***. (10)

26.2 Determine the median of the distribution. (The median is defined as the value ***m*** such that ***P*(*X* ≤*m*) = 0,5**.) Accept $a = \frac{1}{18}$. (12)

[22]

QUESTION 27

Sour worms are sold in packets which have a mass that is normally distributed about a mean of 250 g and a standard deviation of 6 g.

In a box which contains 100 packets, calculate how many are likely to weigh less than 240 g or more than 260 g. **[14]**

QUESTION 28

A certain newspaper predicts that 53% of the population in South Africa would like the capital city of Pretoria to change its name to Tshwane.

28.1 What size sample was necessary to get this estimate correct to within 5% with 98% confidence? (10)

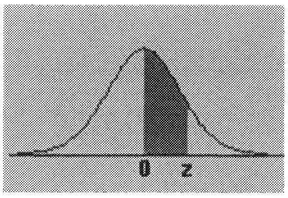
28.2 If I had surveyed 2 000 people, what would the accuracy of my prediction have been for the same degree of confidence? Give the answer as a percentage, correct to one decimal place. (6)

[16]

TOTAL FOR SECTION E: [100]

TOTAL: 400

Normal Distribution/ *Normaalverdeling*



$$P(X \leq x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e \frac{-x^2}{2_{dx}}$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0		0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

FORMULA SHEET/ *FORMULEBLAD*

Differential and Integral Calculus

Differensiaal- en Integraalrekene

$s = r\theta$ $A = \frac{1}{2}r^2\theta$

$\sin^2 x = \frac{1}{2}(1-\cos 2x)$ $\cos^2 x = \frac{1}{2}(1+\cos 2x)$

$\sin A.\cos B = \frac{1}{2}(\sin(A+B)+\sin(A-B))$

$\sin A.\sin B = \frac{1}{2}(\cos(A-B) - \cos(A+B))$

$\cos A.\cos B = \frac{1}{2}(\cos(A-B)+\cos(A+B))$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(2n+1)(n+1)}{6}$$

$$a_{n+1} = a_n - \frac{f(a_n)}{f'(a_n)}$$

$$V = \pi \int_a^b [f(x)]^2 dx$$

$$\text{Riemann Sum} = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x_i$$

F(x)	F '(x)
$a.x^n$	$na.x^{n-1}$
sin x	cos x
cos x	- sin x
tan x	$\sec^2 x$
sec x	sec x.tan x
cot x	$-\operatorname{cosec}^2 x$
cosec x	- cosec x.cot x
arc sin x bg sin x	$\frac{1}{\sqrt{1-x^2}}$
arc cos x bg cos x	$-\frac{1}{\sqrt{1-x^2}}$
arc tan x bg tan x	$\frac{1}{x^2+1}$
f(x).g(x)	f '(x).g(x)+f(x).g '(x)
$\frac{f(x)}{g(x)}$	$\frac{f'(x).g(x)-f(x).g'(x)}{[g(x)]^2}$
f(g(x))	f '(g(x)).g '(x)

Finance/ *Finansies*

$F=P(1+i)^n$ $F=P(1-i)^n$

$F=P(1+in)$ $F=P(1-in)$

$$P = x.\frac{1-(1+i)^{-n}}{i} \qquad F = x.\frac{(1+i)^n - 1}{i}$$

Analytical Geometry/ *Analitiese Meetkunde*

$y=4ax^2$ $yy_1=2a(x+x_1)$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad \frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$$

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad \frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1$$

Algebra

$$\alpha+\beta=-\frac{b}{a} \qquad \alpha+\beta+\gamma=-\frac{b}{a}$$

$$\alpha.\beta=\frac{c}{a} \qquad \alpha\beta+\beta\gamma+\alpha\gamma=\frac{c}{a}$$

$$\alpha.\beta.\gamma=-\frac{d}{a}$$

Statistics / *Statistiek*

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$_nP_r = \frac{n!}{(n-r)!} \qquad _nC_r = \frac{n!}{(n-r)!r!}$$

$$P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$P(X=x) = \frac{\binom{p}{x}\binom{N-p}{n-x}}{\binom{N}{n}}$$

$$z = \frac{\overline{X}-\mu}{\sigma}$$

$$P(\overline{X} - 1.96\frac{\sigma}{\sqrt{n}} < \mu < \overline{X} + 1.96\frac{\sigma}{\sqrt{n}}) = 0.95$$

$$P\left(p - 1.96\sqrt{\frac{p(1-p)}{n}} < \pi < p + 1.96\sqrt{\frac{p(1-p)}{n}}\right) = 0,95$$

Mathematics Formula Sheet / *Wiskunde Formuleblad*

1. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
2. $T_n = a + (n-1)d$
3. $S_n = \dfrac{n}{2}(a + l)$
4. $S_n = \dfrac{n}{2}[2a + (n-1)d]$
5. $T_n = ar^{n-1}$
6. $S_n = \dfrac{a(1 - r^n)}{1 - r}$
7. $S_n = \dfrac{a(r^n - 1)}{r - 1}$
8. $S_\infty = \dfrac{a}{1 - r}$
9. $A = P(1 + \dfrac{r}{100})^n$
10. $A = P(1 - \dfrac{r}{100})^n$
11. $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$
12. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
13. $y = mx + c$
14. $y - y_1 = m(x - x_1)$
15. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
16. $m = \tan\theta$
17. $(\dfrac{x_1 + x_2}{2}; \dfrac{y_1 + y_2}{2})$
18. $y^2 + x^2 = r^2$
19. $(x - p)^2 + (y - q)^2 = r^2$
20. $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$
21. $a^2 = b^2 + c^2 - 2bc.\cos A$
22. $area\ \Delta ABC = \frac{1}{2}ab.\sin C$
23. $\cos(A + B) = \cos A.\cos B - \sin A.\sin B$
24. $\sin(A + B) = \sin A.\cos B + \cos A.\sin B$
25. $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$
26. $\cos 2A = \cos^2 A - \sin^2 A$
27. $\sin 2A = 2\sin A\cos A$

END / *EINDE*