

GAUTENG DEPARTMENT OF EDUCATION
GAUTENGSE DEPARTEMENT VAN ONDERWYS

SENIOR CERTIFICATE EXAMINATION
SENIOR SERTIKAAAT-EKSAMEN

ADDITIONAL MATHEMATICS HG
ADDISIONELE WISKUNDE HG

POSSIBLE ANSWERS / MOONTLIKE ANTWOORDE SUPP 2007

SECTION / AFDELING A
COMPULSORY / VERPLIGTEND
CALCULUS

QUESTION / VRAAG 1

1.1 $s = r?$

$$\therefore s = 2 \cdot \frac{\pi}{6}$$

$$\text{circumference / omtrek} = \frac{\pi}{3} k$$

$$\text{area / oppervlakte} = \frac{1}{2} \text{ or } \frac{1}{2} k^2$$

$$= \frac{1}{2} \cdot \frac{\pi}{6} \cdot 4 k$$

$$= \frac{\pi}{3} k$$

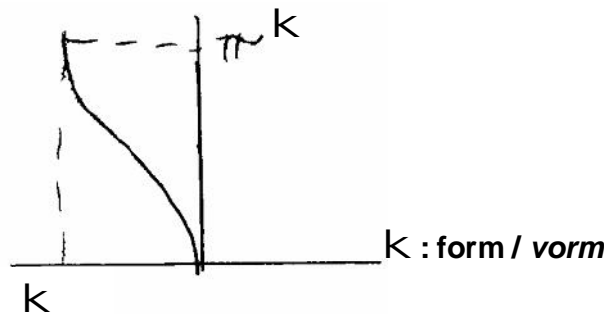
(6)

1.2.1 $-1 = x + 1 = 1 k$

$$\therefore -2 = x = 0 k$$

(4)

1.2.2



(6)

1.3.1 $\arcsin / \text{bg sin} \left(-\sin \frac{\pi}{6} \right)$

$$= \arcsin / \text{bg sin} \left(-\frac{1}{2} \right) k$$

$$= -\frac{\pi}{6} k$$

(6)

$$\begin{aligned}
 1.3.2 \quad & \tan \frac{7\pi}{12} \\
 &= \tan \left(\frac{\pi}{4} + \frac{\pi}{3} \right) \\
 &= \frac{\tan \frac{\pi}{4} + \tan \frac{\pi}{3}}{1 - \tan \frac{\pi}{4} \cdot \tan \frac{\pi}{3}} \\
 &= \frac{1 + \sqrt{3}}{1 - \sqrt{3}}
 \end{aligned}$$

(6)
[28]

QUESTION / VRAAG 2

$$\begin{aligned}
 2.1.1 \quad & \lim_{x \rightarrow 2^-} f(x) = 4 \quad \text{and/en} \quad \lim_{x \rightarrow 2^+} f(x) = 4 \\
 & f(2) = 5 \\
 & \therefore \text{Not continuous} - \text{removable} / \text{Nie kontinu nie - verwyderbaar}
 \end{aligned}$$

(8)

$$\begin{aligned}
 2.1.2 \quad & \lim_{x \rightarrow 4^-} f(x) = 2 \quad \text{and/en} \quad \lim_{x \rightarrow 4^+} f(x) = 2 \\
 & f(4) = 2 \\
 & \therefore \text{Continuous at / kontinuerlik by } x = 4
 \end{aligned}$$

(6)

$$\begin{aligned}
 2.2 \quad & \lim_{x \rightarrow 4^-} f'(x) = -1 \quad \text{and/en} \quad \lim_{x \rightarrow 4^+} f'(x) = 0 \\
 & \therefore \text{Not differentiable} / \text{Nie differensieerbaar nie}
 \end{aligned}$$

(6)
[20]

QUESTION / VRAAG 3

$$\begin{aligned}
 3.1 \quad & \lim_{x \rightarrow \pi/4} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} \quad \left(\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}; \therefore \begin{smallmatrix} L \\ H \end{smallmatrix} \right) \\
 &= \lim_{x \rightarrow \pi/4} \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x - \sin x} \quad \lim_{x \rightarrow \pi/4} \frac{-2\sin 2x}{-\sin x - \cos x} \\
 &= \lim_{x \rightarrow \pi/4} (\cos x + \sin x) \quad = \frac{-2}{-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}} \\
 &= 2 \cdot \frac{1}{\sqrt{2}} \quad (\text{or/of } \sqrt{2}) \quad (\text{or / of } 1,4) \quad = \sqrt{2} \quad (8)
 \end{aligned}$$

$$\begin{aligned}
 3.2 \quad & \lim_{x \rightarrow \infty} \frac{x \left(\frac{2}{x} - 1 \right)}{x \sqrt{9 + \frac{1}{x^2}}} \\
 &= -\frac{1}{3} \quad (6) \\
 & \quad [14]
 \end{aligned}$$

QUESTION / VRAAG 4

$$\begin{aligned}
 4.1 \quad & f(x+h) - f(x) = \sqrt{2x+2h-1} - \sqrt{2x-1} \\
 & f'(x) = \lim_{h \rightarrow 0} \frac{\sqrt{2x+2h-1} - \sqrt{2x-1}}{h} \cdot \frac{\sqrt{2x+2h-1} + \sqrt{2x-1}}{\sqrt{2x+2h-1} + \sqrt{2x-1}} \\
 &= \lim_{h \rightarrow 0} \frac{2x+2h-1-2x+1}{h(\sqrt{2x+2h-1} + \sqrt{2x-1})} \\
 &= \frac{2}{\sqrt{2x-1} + \sqrt{2x-1}} \\
 &= \frac{1}{\sqrt{2x-1}} \quad (10)
 \end{aligned}$$

$$4.2 \quad \frac{dy}{dx} = \frac{-\csc 3x \cdot \cot 3x \cdot 3x^3 - \csc 3x \cdot 3x^2}{x^6} \quad (8)$$

$$4.3 \quad \frac{3\sin^2(\arctan/bg \tan x) \cdot \cos(\arctan/bg \tan x)}{1+x^2} \quad (8)$$

$$\begin{aligned}
4.4 \quad f(x) &= 2(1 + 2x)^{-1} \quad \text{K} \\
f'(x) &= -2^2(1 + 2x)^{-2} \quad \text{K} \\
f''(x) &= 2 \cdot 2^3(1 + 2x)^{-3} \\
f'''(x) &= -3 \cdot 2 \cdot 2^4(1 + 2x)^{-4} \\
f^{(n)}(x) &= (-1)^n \cdot n! 2^{n+1} (1 + 2x)^{-n-1} \quad \text{K} \\
&\quad \text{K} \quad \text{K} \quad \text{K}
\end{aligned}
\tag{12}$$

[38]

QUESTION / VRAAG 5

$$? \quad xi = \frac{3}{n} \quad \text{K}$$

$$xi = 3 + \frac{3i}{n} \quad \text{K}$$

$$\begin{aligned}
f(xi) &= \left(3 + \frac{3i}{n}\right)^2 - 4\left(3 + \frac{3i}{n}\right) \quad \text{K} \\
&= 9 + \frac{18i}{n} + \frac{9i^2}{n^2} - 12 - \frac{12i}{n} \\
&= \frac{9i^2}{n^2} + \frac{6i}{n} - 3 \quad \text{K}
\end{aligned}$$

$$f(xi) \cdot \Delta xi = \frac{27i^2}{n^3} + \frac{18i}{n^2} - \frac{9}{n} \quad \text{K}$$

$$\begin{aligned}
\sum_{i=1}^n f(xi) \cdot \Delta xi &= \sum_{i=1}^n \left(\frac{27i^2}{n^3} + \frac{18i}{n^2} - \frac{9}{n} \right) \quad \text{K} \\
&= \frac{27}{n^3} \cdot \frac{n}{6} (n+1)(2n+1) + \frac{18}{n^2} \left(\frac{n}{2} \right) (n+1) - \frac{9}{n} \cdot n \quad \text{K}
\end{aligned}$$

$$\begin{aligned}
\lim_{n \rightarrow \infty} \sum_{i=1}^n f(xi) \cdot \Delta xi &= 9 + 9 - 9 \\
&= 9 \quad \text{K}
\end{aligned}$$

[20]

QUESTION / VRAAG 6

$$6.1 \quad \frac{k}{2} (\frac{k}{2} \tan 2x)^2 + k \quad (6)$$

$$\begin{aligned}
 6.2 \quad & k \int \frac{dx}{4 \left(\left(\frac{3x}{2} \right)^2 + 1 \right)} \\
 &= \frac{1}{6} \arctan \frac{\frac{k}{2} \tan \frac{3x}{2}}{\frac{2}{\sqrt{3}}} = \frac{1}{6} \arctan \frac{\frac{k}{2} \tan \frac{3x}{2}}{\frac{2}{\sqrt{3}}} + C \\
 &= \frac{1}{4} \arctan \frac{\frac{k}{2} \tan \frac{3x}{2}}{\sqrt{3}} \\
 &= \frac{1}{6 \cdot 3} = \frac{k}{18} \text{ (of 0,1745)} \quad (12) \\
 & \quad [18]
 \end{aligned}$$

QUESTION / VRAAG 7

7.1 At a_1 the tangent to the curve cuts the x-axis at a_2 . / By a_1 sny die raaklyn aan die kromme die x-as by a_2 .
 At a_2 the tangent to the curve cuts the x-axis at a_1 . / By a_2 sny die raaklyn aan die kromme die x-as by a_1 .
 \therefore the values oscillate and don't converge. / die waardes ossileer en konvergeer nie. (interpret answer / interpreteer an two ord – dieselfde woorde nie nodig) (6)

7.2 At b_1 the tangent to the curve is horizontal and will never cut the x-axis again. / By b_1 is die raaklyn by die kromme horisontaal en sny glad nie die x-as weer nie. (Memo open for any feasible answer). (4)

$$\begin{aligned}
 7.3 \quad & f'(x) = -6x^2 + 4x + 2 \quad k \\
 & A_{n+1} = A_n - \left(\frac{2a_n^3 + 2a_n^2 + 2a_n - 1}{-6a_n^2 + 4a_n + 2} \right) \quad k \\
 & C_1 = 0,5 \quad k \\
 & C_2 = 0,4 \quad k \\
 & C_3 = 0,4030 \quad k \\
 & C_4 = 0,4030 \\
 & \therefore P \approx 0,403 \quad k \quad (10) \\
 & \quad [20]
 \end{aligned}$$

$$\begin{aligned}
 8.2 \quad V &= \pi \int_0^{3\pi/4} (\sin x + \cos x)^2 dx \\
 &= \pi \int_0^{3\pi/4} (\sin^2 x + 2\sin x \cos x + \cos^2 x) dx \\
 &= \pi \int_0^{3\pi/4} (1 + 2\sin x \cos x) dx \\
 &= \pi \int_0^{3\pi/4} (1 + \sin 2x) dx \\
 &= \pi \left(x - \frac{1}{2} \cos 2x \right) \Big|_0^{3\pi/4} \\
 &= \pi \left[\left(\frac{3\pi}{4} - \frac{1}{2} \cos \frac{3\pi}{2} \right) - \left(0 - \frac{1}{2} \cos 0 \right) \right] \\
 &= \pi \left(\frac{3\pi}{4} - 0 + \frac{1}{2} \right) \quad \text{OR/OR} \quad \frac{3\pi^2}{4} + 2? \quad \text{OR/OR} \quad 8,9729
 \end{aligned}$$

QUESTION / VRAAG 9

$$9.1 \quad y + 4x = 78 \\ y = 78 - 4x \quad \text{K}$$

$$V = x^2 y \\ = x^2 (78 - 4x) \quad \text{K} \\ = 78x^2 - 4x^3 \quad (4)$$

$$9.2 \quad \frac{dv}{dx} = 156x - 12x^2 = 0 \quad \text{K} \quad \frac{d^2v}{dx^2} = 156 - 24x \quad \text{K} \\ x(13-x) = 0 \quad \text{if/as } x = 13: \\ x = 0 \text{ or/of } x = 13 \quad \text{K} \quad 156 - 24(13) < 0 \\ \therefore y = 78 - 4(13) \quad \therefore \text{max at / maks by } x = 13 \\ = 26 \quad \text{K} \\ \therefore \text{dimensions / afmetings } x = 13 \\ y = 26 \quad (12)$$

[16]

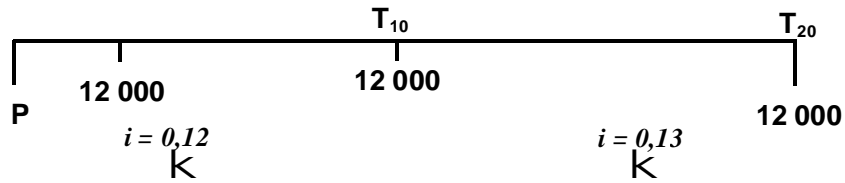
TOTAL FOR SECTION A / TOTAAL VIR AFDELING A: [200]

SECTION / AFDELING B
FINANCIAL MATHEMATICS / WISKUNDE VAN FINANSIES
QUESTION / VRAAG 10

$$P(q) = \frac{-2}{3} q^3 + 30q^2 + 3600q - 15000 \quad \text{K} \\ P'(q) = -2q^2 + 60q + 3600 = 0 \quad \text{K} \\ \therefore q^2 - 30q - 1800 = 0 \\ (q - 60)(q + 30) = 0 \quad \text{K} \\ q = 60 \quad \text{K} \\ P(60) = R165000 \quad \text{K}$$

[14]

QUESTION / VRAAG 11



$$P = x \frac{(1 - (1+i)^{-n})}{i} \text{ K}$$

$$\therefore P = x \frac{(1 - 1,12^{-10})}{0,12} \text{ K} + x \frac{(1 - 1,13^{-10})}{0,13} (1,12)^{-10} \text{ K} \quad \text{or / of}$$

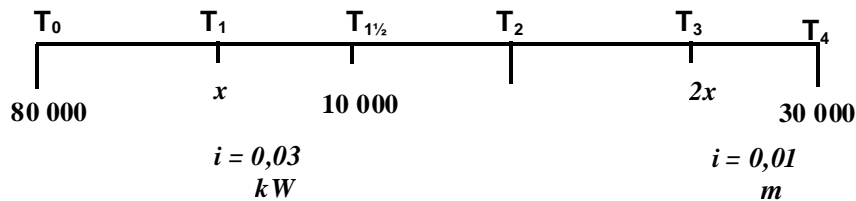
$$P = x(1,12)^{-1} \left(\frac{1 - 1,12^{-10}}{1 - 1,12^{-1}} \right) + x(1,12)^{-10} (1,13)^{-1} \left(\frac{1 - 1,13^{-10}}{1 - 1,13^{-1}} \right)$$

(12)

[12]

QUESTION / VRAAG 12

12.1



$$T_2: 80\,000 (1,03)^8 = x(1,03)^4 + 10\,000 (1,03)^2 + 2x(1,01)^{-12} + 30\,000 (1,01)^{-24}$$

$$x(2,900 \dots) \text{ K} = 67\,105,62$$

$$x = R23\,136,62 \text{ K}$$

Marks: 0,03 K

0,01 K

equation K

months K

quart K

[14]

QUESTION / VRAAG 13

$$13.1 \quad 50\,000 = 190\,734,86 (1 - i)^6$$

$$i = 0,20$$

$$\therefore = 20\%$$

(4)

$$13.2 \quad 190\,734,86 (1,07)^6$$

$$= R286\,241,59$$

(4)

$$13.3.1 \quad (1+i) = 1,01^{12} \quad (4)$$

$$i = 0,1268\% \quad 12,68\%$$

$$13.3.2 \quad F = \frac{4000(1,1268^5 - 1)}{0,1268} (1,1268) = R29\,022,92 \quad (10)$$

$$13.3.3 \quad 265\,266,60 = x \frac{(1,01^{72} - 1)}{0,01} \quad \text{form}$$

$$x = R2\,533,35$$

(Accept / Aanvaar 2 533,34) (12)

[34]

QUESTION / VRAAG 14

14.1

$n = 116 - 5 + 1 = 112$

$$F = \frac{1000(1,005^{112} - 1)}{0,005} (1,005)^{112}$$

$$= R152\,661,73 \quad (14)$$

14.3

$$152\,661,73 = 3\,000 \frac{(1 - 1,0075^{-n})}{0,0075}$$

$$-n = \frac{\log 0,6183 \dots}{\log 1,0075} \quad \text{K - logs}$$

$$n = 64,3 \dots$$

$$\therefore 64 + 1 \text{ last} \quad (12)$$

[26]

TOTAL FOR SECTION B / TOTAAL VIR AFDELING B: [100]

SECTION / AFDELING C
ANALYTICAL GEOMETRY / ANALITIESE MEEKUNDE

QUESTION / VRAAG 15

$$\begin{aligned}
 15.1 \quad l_1: x + y - 4 &= 0 & l_2: x + 7y - 7 &= 0 \\
 y &= -x + 4 & 7y &= -x + 7 \\
 \tan x &= \frac{-1 - \left(\frac{-1}{7}\right)}{1 + (-1)\left(\frac{-1}{7}\right)} & y &= \frac{-1}{7}x + 1 \\
 &= -\frac{3}{4} \\
 x &= 180 + (-36, 87^\circ) \\
 &= 143,13^\circ
 \end{aligned} \tag{8}$$

$$\begin{aligned}
 15.2 \quad -x - y + 4 &= 0 \quad (1) \\
 -x - 7y + 7 &= 0 \quad (2) \\
 \frac{-x - y + 4}{\sqrt{2}} &= \frac{-x - 7y + 7}{\sqrt{50}} \\
 5(-x - y + 4) &= -x - 7y + 7 \\
 -5x - 5y + 20 &= -x - 7y + 7 \\
 -4x + 2y + 13 &= 0
 \end{aligned} \tag{8}$$

[16]

QUESTION / VRAAG 16

$$\begin{aligned}
 16.1 \quad x^2 + y^2 - 8x - 4y &= 0 \quad (1) \\
 x^2 + y^2 - 10x + 20 &= 0 \quad (2) \\
 (2) - (1) \quad -2x + 4y + 20 &= 0 \\
 2y + 10 &= x
 \end{aligned} \tag{4}$$

16.2 *Subst for / Subst. vir x in (1)*

$$(2y + 10)^2 + y^2 - 8(2y + 10) - 4y = 0$$

$$4y^2 + 40y + 100 + y^2 - 16y - 80 - 4y = 0$$

$$5y^2 + 20y + 20 = 0$$

$$y^2 + 4y + 4 = 0$$

$$(y+2)^2 = 0$$

$$y = -2$$

\therefore One point of intersection only / Slegs een raakpunt

\therefore Common tangent / Gemeenskaplike raaklyn

(8)

[12]

QUESTION / VRAAG 17

17.1 $y^2 + 8x - 6y + 1 = 0$

$$y^2 - 6y + 9 = -8x - 1 + 9$$

$$(y - 3)^2 = -8x + 8$$

$$= -8(x - 1)$$

$$= 4(-2)(x - 1)$$

(8)

17.2.1 $(1 ; 3)$

(2)

17.2.2 $(-1 ; 3)$

(2)

17.2.3 $x = 3$

(2)

17.2.4 $|4(-1)|$

$$= 4$$

(2)

[16]

QUESTION / VRAAG 18

18.1 $9x^2 - 16y^2 + 54x - 32y = 79$

$$9(x^2 + 6x) - 16(y^2 + 2y) = 79$$

$$9(x^2 + 6x + 9) - 16(y^2 + 2y + 1) = 79 + 81 - 16$$

$$9(x+3)^2 - 16(y+1)^2 = 144$$

$$\frac{9(x+3)^2}{144} - \frac{16(y+1)^2}{144} = 1$$

$$\frac{(x+3)^2}{16} - \frac{(y+1)^2}{9} = 1$$

(10)

18.2 $\frac{x+3}{4} + \frac{y+1}{3} = 0$

$$3x + 9 + 4y + 4 = 0$$

$$3x + 4y + 13 = 0$$

$$4y = -3x - 13$$

$$y = \frac{-3x - 13}{4}$$

$$\frac{x+3}{4} - \frac{y+1}{3} = 0$$

$$3x + 9 - (4y + 4) = 0$$

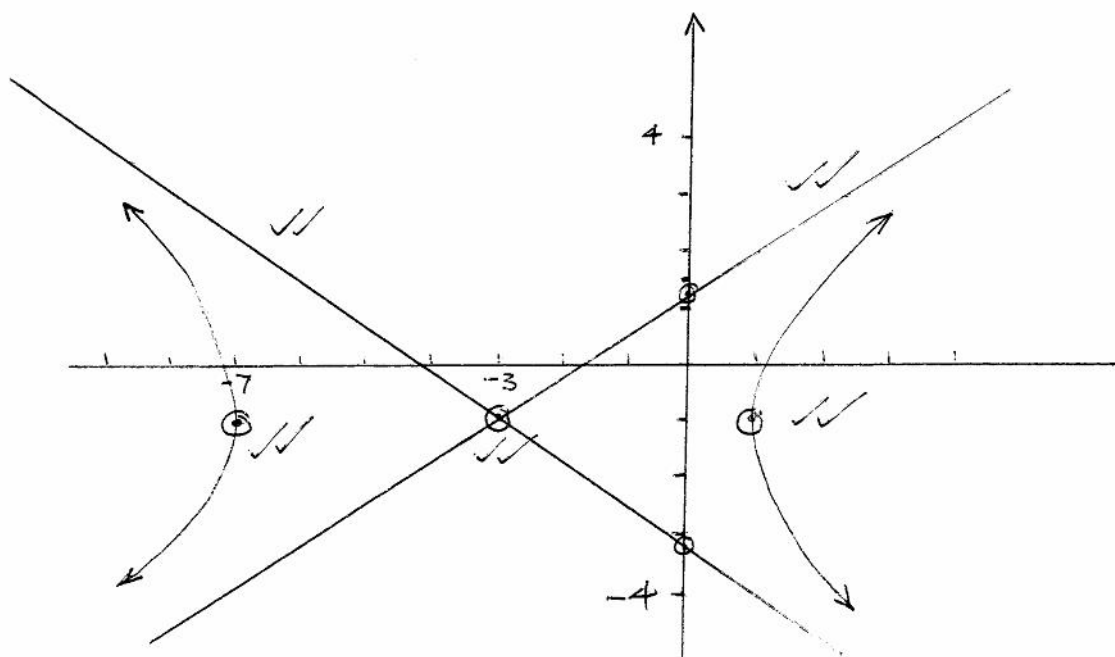
$$3x + 4y + 5 = 0$$

$$4y = 3x - 5$$

$$y = \frac{3x - 5}{4}$$

(8)

18.3 Centre / Middel (-3; -1)
Vertices/ Snypunte (1; -1) (-7; -1)



(10)

[28]

QUESTION / VRAAG 19

19.1 $AC: a_1 = 1 \quad b_1 = 2 \quad c_1 = -1$
 $BC: a_2 = 2 \quad b_2 = -1 \quad c_2 = -3$

$$AC = \sqrt{1^2 + 2^2 + (-1)^2} \quad BC = \sqrt{2^2 + (-1)^2 + (-3)^2}$$

$$= \sqrt{6} \quad = \sqrt{14}$$

$$\cos \theta = \frac{(1)(2) + (2)(-1) + (-1)(-3)}{\sqrt{6} \cdot \sqrt{14}}$$

$$= \frac{2 + (-2) + 3}{\sqrt{84}}$$

$$= \frac{3}{\sqrt{84}}$$

$$= 0,327$$

$$\theta = 70,9^\circ \quad (12)$$

19.2 $Area/Opp. = \frac{1}{2} (AC)(BC) \sin \theta$

$$= \frac{1}{2} \sqrt{6} \sqrt{14} \sin 70,9^\circ$$

$$= 4,33 \quad (4)$$

19.3 a, b & c are direction numbers of line l_1 , normal to plane.
 a, b & c is rigtingno mmer s vir lyn l_1 , normaal op vlak

$$(1) a + (2) b + (-1) c = 0 \quad \text{i.e.} \quad a + 2b - c = 0 \quad (1)$$

$$(2) a + (-1) b + (-3) c = 0 \quad a - b - 3c = 0 \quad (2)$$

$$(1) - (2) \quad 3b + 2c = 0$$

$$c = -\frac{3b}{2}$$

Sub in (1)

$$a + 2b - \frac{3b}{2} = 0$$

$$a = -\frac{b}{2}$$

If / As $b = 2$; $a = -1$; $c = -3$

$$-1(x-1) + 2(y-0) + (-3)(Z-2) = 0$$

$$-x + 1 + 2y - 3Z + 6 = 0$$

$$-x + 2y - 3Z + 7 = 0 \quad (12)$$

[28]

TOTAL FOR SECTION C / TOTAAL VIR AFDELING C: [100]

SECTION / AFDELING D

ALGEBRA

QUESTION / VRAAG 20

20.1 Let / Laat $n = 1$:

$$LHS = \frac{1}{2} \quad \text{RHS} = 1 - \frac{1}{2} = \frac{1}{2}$$

\therefore Statement true for / Stelling waar vir $n = 1$

Assume statement true for / Aanvaar stelling waar $n = k$:

$$\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots + \frac{1}{2^k} = 1 - \frac{1}{2^k}$$

Let / Laat $n = k + 1$

$$LHS = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots + \frac{1}{2^k} + \frac{1}{2^{k+1}}$$

$$= 1 - \frac{1}{2^k} + \frac{1}{2^{k+1}}$$

$$= 1 - \frac{1}{2^k} + \frac{1}{2} \cdot \frac{1}{2^k} \quad \text{K}$$

$$= 1 - \frac{1}{2} \cdot \frac{1}{2^k} \quad \text{K}$$

$$= 1 - \frac{1}{2^{k+1}} = RHS$$

\therefore If statement is true for $n = k$, it is also true for $n = k+1$.

As stelling waar is vir $n = k$, is dit ook waar vir $n = k + 1$

\therefore True / Waar $\forall n \in \mathbb{N}$. K

(16)

$$\begin{aligned}
 20.2 \quad & \frac{4}{(x^2+2)(x^2+1)} = \frac{Ax+B}{x^2+2} + \frac{Cx+D}{x^2+1} \\
 & 4 = (Ax+B)(x^2+1) + (Cx+D)(x^2+2) \\
 & x^3: 0 = A + C \quad \text{--- j --- k} \\
 & x^2: 0 = B + D \quad \text{--- k --- k} \\
 & x: 0 = A + 2C \quad \text{--- l --- k} \\
 & C: 4 = B + 2D \quad \text{--- m --- k} \\
 & \text{m-k: } 4 = D \quad B = -4 \quad \text{--- k} \\
 & \text{l-j: } C = 0 \quad A = 0 \quad \text{--- k} \\
 & \frac{-4}{x^2+2} + \frac{4}{x^2+1}
 \end{aligned} \tag{16}$$

$$\begin{aligned}
 20.3 \quad & a + b + c = 3 \quad \text{--- k} \\
 & ab + ac + bc = 2 \quad \text{--- k} \\
 & abc = 1 \\
 & (a + b + c)(a + b + c) \\
 & = a^2 + ab + ac + ab + b^2 + bc + ac + bc + c^2 \quad \text{--- k} \\
 & \therefore a^2 + b^2 + c^2 = (a + b + c)^2 - 2(ab + ac + bc) \quad \text{--- k} \\
 & = 9 - 2 \cdot 2 \quad \text{--- k} \\
 & = 5 \quad \text{--- k}
 \end{aligned} \tag{14}$$

[46]

QUESTION / VRAAG 21

- 21.1 If there is a polynomial / Indien daar ? polinoom $P(x) \in \mathbb{Z}[x]$ is / and there exists a prime number p such that / en daar ? priemgetal p bestaan sodat
- (i) the main coefficient is not divisible by p / die hoofkoeffisiënt nie deur p deelbaar is nie. k
 - (ii) all other coefficients are divisible by p / al die ander koëffisiënte deelbaar is deur p k
 - (iii) the constant is not divisible by p^2 then $P(x)$ is irreducible in $\mathbb{Q}[x]$ {or $\mathbb{Z}[x]$ } / is die konstante n nie deelbaar deur p^2 nie en dan is $P(x)$ onontbindbaar in $\mathbb{Q}[x]$ {of $\mathbb{Z}[x]$ } k
- (8)

21.2 Let / Laat $p = 3$: \mathbb{K}

- 1 not div. by 3 / 1 is nie deur 3
 - 6, 18, 12 div. by 3 / 6, 18, 12. deur 3
 - 12 not div by 3^2 / 12 nie deelbaar deur 3^2
- \therefore irreducible in $\mathbb{Z}[x]$ / onontbindbaar deur $\mathbb{Z}[x]$ \mathbb{K} } m

(8)

21.3 $1 + \sqrt{2}$ also a zero / ook ? nul \mathbb{K}

$(x - 1 + \sqrt{2})(x - 1 - \sqrt{2}) \stackrel{\mathbb{K}}{=} \text{a factor / ? faktor}$

$$= x^2 - 2x - 1 \quad \mathbb{K}$$

$$\begin{array}{r} 1 - 2 - 1 \overline{) 1 - 1 - 5 \quad 5} \\ \underline{1 - 3 - 4 \quad 16 - 5 - 5} \\ 1 - 2 - 1 \\ \underline{-1 - 3 \quad 16} \\ -1 \quad 2 \quad 1 \quad \mathbb{K} \\ \underline{-5 \quad 15 \quad -5} \\ -5 \quad 10 \quad 5 \\ \underline{5 \quad -10 \quad -5} \\ 5 \quad -10 \quad -5 \end{array}$$

$$f(x) = (x^2 - 2x - 1) \stackrel{\mathbb{K}}{=} (x^3 - x^2 - 5x + 5) = (x^2 - 2x - 1) \stackrel{\mathbb{K}}{=} (x^2 - 5)(x + 1)$$

$$= \underbrace{(x - 1 + \sqrt{2})}_{\mathbb{K}} \underbrace{(x - 1 - \sqrt{2})}_{\mathbb{K}} \underbrace{(x - \sqrt{5})}_{\mathbb{K}} \underbrace{(x + \sqrt{5})}_{\mathbb{K}} \underbrace{(x - 1)}_{\mathbb{K}}$$

(18)

[34]

QUESTION / VRAAG 22

22.1 $x^2 - 4x - 5 = 0$

$$(x - 5)(x + 1) = 0 \quad \mathbb{K}$$

$$x = 5 \text{ or / of } x = -1 \quad \mathbb{K}$$

$$y = 5 \quad \mathbb{K}$$

(6)

22.2 Vertical / Vertikaal: $x = \frac{1}{2} \quad \mathbb{K}$

Horisontal: none / Horisontaal: geen

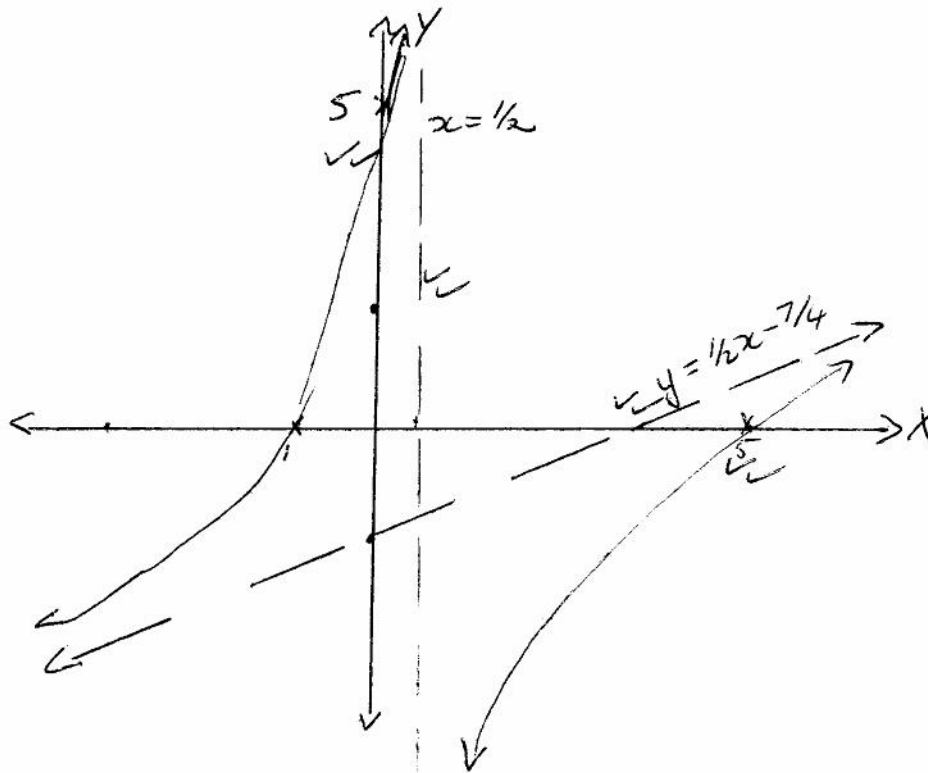
Oblique / Skui ns: $\frac{1}{2} \quad -\frac{7}{4}$

$$\begin{array}{r} 2 - 1 \overline{) 1 - 4 - 5} \\ \underline{1 - \frac{1}{2}} \\ -3\frac{1}{2} - 5 \\ \underline{-3\frac{1}{2} \quad +\frac{7}{4}} \end{array}$$

$$\therefore \bar{\mathbb{K}} = \frac{1}{2}x - \frac{7}{4} \quad \mathbb{K}$$

(6)

22.3



(8)

[20]

TOTAL FOR SECTION D / TOTAAL VIR AFDELING D: [100]

SECTION / AFDELING E
STATISTICS / STATISTIEK
QUESTION / VRAAG 23

23.1.1 $18^3 = 5\,832$ K (4)

23.1.2 $18 \cdot 17 \cdot \binom{K}{6} = 2450448$ or ${}^{18}P_2 \cdot {}^{16}C_6$ or ${}^{18}C_6 \cdot {}^{16}P_2$ (6)

23.2 $2 \cdot \binom{K}{2} = 182$ K (6)

[16]

QUESTION / VRAAG 24

$$P(A) \cdot P(B) = P(A \cap B) \quad \text{K}$$

$$P(A) = 2P(B)$$

$$\therefore P(A \cup B) = 0,625$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad \text{K}$$

$$0,625 = 2P(B) + P(B) - 2P(B) \cdot P(B) \quad \text{K}$$

$$2P(B)^2 - 3P(B) + 0,625 = 0 \quad \text{K}$$

$$\text{K } (2P(B) - 2,5)(P(B) - 0,25) = 0 \quad \text{K}$$

$$P(B) = 0,25 \quad \text{K}$$

$$P(A) = 0,5 \quad \text{K}$$

$$\text{or } 16(P(B))^2 - 24P(B) + 5 = 0$$

$$(4P(B) - 1)(4P(B) - 5) = 0$$

$$P(B) = 1/4$$

$$P(A) = 1/2$$

(16)

[16]

QUESTION / VRAAG 25

$$25.1 \quad \binom{6}{2} 0,15^2 \cdot 0,85^4 = 0,1762$$

(8)

$$25.2 \quad P(X \geq 2) = 1 - P(X = 0,1)$$

$$= 1 - \left(\binom{6}{0} 0,15^0 \cdot 0,85^6 - \binom{6}{1} 0,15^1 \cdot 0,85^5 \right)$$

$$= 0,2235 \quad \text{K}$$

(8)

[16]

QUESTION / VRAAG 26

$$26.1 \quad \int_0^6 a(6-x) dx = 1 \quad \text{K}$$

$$\left(6ax - \frac{1}{2}ax^2 \right) \Big|_0^6 = 1$$

$$36a - 18a = 1$$

$$18a = 1$$

$$a = 1/18 \quad \text{K}$$

(10)

$$26.2 \quad \int_0^m \frac{1}{18}(6-x)dx = 0,5 \quad \text{K}$$

$$\left(\frac{x^2}{3} - \frac{x^2}{36} \right) \Big|_0^m = 0,5$$

$$12m - m^2 = 18 \quad \text{K}$$

$$m^2 - 12m + 18 = 0$$

$$m = \frac{12 \pm \sqrt{72}}{2} \quad \text{K}$$

$$m = \frac{10,2}{\text{n.a.}} \quad \text{or / of} \quad m = 1,7574 \quad \text{K}$$

(12)

[22]

QUESTION / VRAAG 27

$$\begin{aligned} P(X < 240) + P(X > 260) \\ &= P(Z < -1,67) + P(Z > 1,67) \\ &= 2(0,5 - 0,4525) \\ &= 0,095 \end{aligned}$$



$$\therefore 0,095 \times 100 = 9,5 \text{ pac kets / pakk ies}$$

(14)

[14]

QUESTION / VRAAG 28

$$\begin{aligned} 28.1 \quad & 2,33 \sqrt{\frac{0,53 \cdot 0,47}{n}} < 0,05 \\ & n > 540,9 \\ & \therefore 541 \text{ people / mense} \end{aligned}$$

(10)

$$\begin{aligned} 28.2 \quad & 2,33 \sqrt{\frac{0,53 \cdot 0,47}{2000}} \quad \text{K} \\ & = 0,0260 \quad \text{K} \\ & = 2,6 \% \quad \text{K} \end{aligned}$$

(6)

[16]

TOTAL FOR SECTION E / TOTAAL VIR AFDELING E: [100]

TOTAL / TOTAAL: 400