THE INSTITUTION OF ENGINEERS, SRI LANKA

PART I EXAMINATION - MARCH 2006

MATHEMATICS **Time allowed: Three**

Answer FIVE questions only.

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1. Given a 3×3 matrix $A=\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$ define its

   (i) determinant **det** A

   (ii) adjoint **adj** A

Verify that **adj** $A\cdot A$ = $A\cdot$ **adj** A = **det** $A\cdot I_3$ where I_3 denotes the 3×3 identity matrix.

If A and B are two 3×3 matrices and λ is a real constant, state <u>without proof</u> whether each of the following statements is **<u>true</u>** or **<u>false</u>**:

(i) **det**$(A\cdot B)$ = **det** $A\cdot$ **det** B

(ii) **adj**$(A\cdot B)$ = **adj** $A\cdot$ **adj** B

(iii) **det** $(\lambda\cdot B)$ = $\lambda\cdot$ **det** B

(iv) **adj**$(\lambda\cdot B)$ = $\lambda\cdot$ **adj** B

2 Find the values of p for which the system of linear equations $\begin{aligned} x+py+p^2z&=8 \\ p^2x-8y+pz&=14 \\ px+p^2y+8z&=0 \end{aligned}$ has

(a) a unique solution

(b) no solution

(c) an infinite number of solutions

For each of the following systems determine all solutions that exist:

(i) $\begin{aligned} x+y+z&=8 \\ x-8y+z&=14 \\ x+y+8z&=0 \end{aligned}$ **(ii)** $\begin{aligned} x+2y+4z&=8 \\ 4x-8y+2z&=14 \\ 2x+4y+8z&=0 \end{aligned}$ **(iii)** $\begin{aligned} x-2y+4z&=8 \\ 4x-8y-2z&=14 \\ -2x+4y+8z&=0 \end{aligned}$

3 State De Moivre's theorem.

If $z=\sqrt{3}+i$, determine the modulus and arguments of the complex numbers z, $8\sqrt{3}\,z$ and z^4.

Indicate in an Argand diagram the points representing these complex numbers.

Hence, deduce that $z=\sqrt{3}+i$ is a solution of the equation: $z^4-8\sqrt{3}\,z+32=0$

What are the other roots of this equation?.

4 $\underline{a}=-3\underline{i}+2\underline{j}-5\underline{k}$, $\underline{b}=-3\underline{i}+6\underline{j}-8\underline{k}$, $\underline{c}=-5\underline{i}-\underline{j}+\underline{k}$ and $\underline{d}=-2\underline{i}-2\underline{j}-$ [illegible]e position vectors respectively of the four vertices A, B, C and D of a tetrahedron. [illegible]e the perpendicular distance from the point A to the plane passing through B, C and D.

Forces of magnitude 15, 14 and 9 act along the sides AB, AC and AD respectively. Prov[illegible] the line of action of the resultant force passes through the origin.

5. **(a)** Determine the limit: $\lim\limits_{x\to 0}\left(\dfrac{x\sin 3x}{1-\cos 4x}\right)$

(b) Find the maximum and minimum values of the function $f(x)=e^{-2x}(x^3-x+1)$ and draw a rough sketch of its graph.

(c) Use Leibnitz's theorem to evaluate: $\left(\dfrac{d^3}{dx^3}\right)\left(e^{2x}\sin 3x\right)$

6. **(a)** Evaluate the following definite integrals

(i) $\displaystyle\int_0^{\frac{\pi}{4}}\frac{1}{1-\sin x}\,dx$ **(ii)** $\displaystyle\int_1^2\frac{1+x+2x^2}{1-x-2x^2}\,dx$ **(iii)** $\displaystyle\int_0^1 x^3e^{2x}\,dx$

(b) Let A be the region bounded by the curves $x=0$, $x=6$, $y=0$ and $y=x^2-4x+9$ in the first quadrant. Determine

(i) the area of the region A

(ii) the volume of the solid obtained by rotating A around the x-axis.

7. Solve the following initial value problems:

(a) $\dfrac{dy}{dx}-2y=1-2x$; $y(1)=1$

(b) $\dfrac{dy}{dx}=\dfrac{2y^2}{x^2+xy}$; $y(1)=1$

(c) $\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}+2y=2x+4$; $y(0)=1$, $y'(0)=2$

8. **(a)** Using an appropriate numerical method, obtain a solution ation $2\sin x = \cos 2x + 1$ in the interval $(0, 1)$

Obtain the exact solution of the same equation by analytical methods, and hence determ percentage of error in the answer obtained by your numerical method.

(b) Using an appropriate numerical method, evaluate the integral $\int_0^1 \frac{1}{1+\sin x}\,dx$,

Obtain the exact value of the same integral by analytical methods, and hence determine the percentage of error in the answer obtained by your numerical method.

9. **(a)** Using the Jacobi method or the Gauss-Seidel method solve the linear system of equations:

$$\begin{aligned} 4x + y + z &= 8 \\ x - 5y + z &= 1 \\ x + 2y + 8z &= 0 \end{aligned}$$

(b) Given that $\sin\left(\frac{\pi}{3}\right) = 0.8660$, $\sin\left(\frac{\pi}{4}\right) = 0.7071$ and $\sin\left(\frac{\pi}{6}\right) = 0.5$, use three point Lagrangian interpolation formula to calculate $\sin\left(\frac{\pi}{5}\right)$ approximately