



HIGHER SCHOOL CERTIFICATE EXAMINATION

1997
**MATHEMATICS IN
SOCIETY**

2 UNIT

*Time allowed—Two hours and a half
(Plus 5 minutes reading time)*

DIRECTIONS TO CANDIDATES

- Only Board-approved calculators are to be used.
- Show all necessary working for Section II and Section III.
- Marks may be deducted for careless or badly arranged work.
- You may ask for extra Writing Booklets if you need them.
- The mark out of 80 will be converted to a mark out of 100.

Section I (20 marks)

- This Section contains 20 multiple-choice questions.
- Attempt ALL questions.
- All questions are of equal value.
- Mark your answers in pencil on the Answer Sheet provided.
- Allow about 45 minutes for this Section.

Section II (36 marks)

- Attempt ALL questions.
- Each question is worth 12 marks.
- Answer each question in a *separate* Writing Booklet.
- Allow about one hour for this Section.

Section III (24 marks)

- Attempt TWO questions.
- Each question is worth 12 marks.
- Answer each question in a *separate* Writing Booklet.
- Allow about 45 minutes for this Section.

SECTION I

Attempt ALL questions.

All questions are of equal value.

Select the alternative A, B, C, or D that best answers the question.

Mark your answers in pencil on the *separate* Answer Sheet provided.

1. It takes 20 seconds to fill a ten-litre bucket with water.

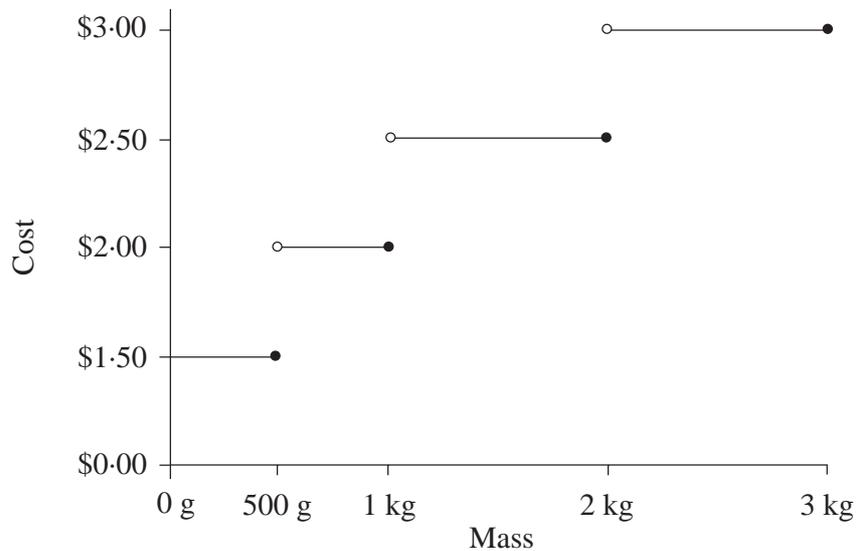
What is the rate of flow in litres per hour?

- (A) 30 (B) 180 (C) 1800 (D) 3600
-

2. Simplify $\left(\frac{t^8}{t^4}\right)^2$.

- (A) t^4 (B) t^6 (C) t^8 (D) t^{12}
-

3. The graph shows the cost of sending parcels of different masses.



Eloise wants to send four parcels each weighing 400 g to her friend.

How much would be saved by sending them together as one parcel, rather than separately?

- (A) \$1.00 (B) \$3.00 (C) \$3.50 (D) \$4.50
-

4. Fiona, Matt, and Mel invested \$30 000, \$50 000, and \$20 000 respectively in a business. In the first year they made a total profit of \$69 000. This was divided in the same ratio as the amounts invested.

How much did Fiona receive?

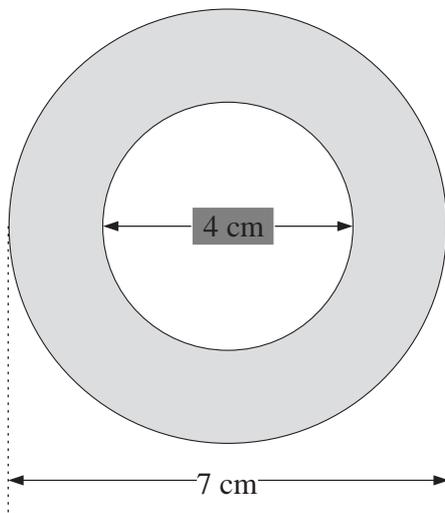
- (A) \$13 800 (B) \$20 700 (C) \$23 000 (D) \$30 000
-

5. A boat sails 3 kilometres west, then 3 kilometres south.

What is its bearing from its original position?

- (A) 045° (B) 135° (C) 225° (D) 315°
-

- 6.



Calculate the area of the shaded part between the two circles.

- (A) 9.42 cm^2
 (B) 25.92 cm^2
 (C) 28.27 cm^2
 (D) 103.67 cm^2
-

7. Use the formula $h = 8 + \frac{18-a}{2}$ to find a if $h = 10$.

- (A) $a = 6$ (B) $a = 7$ (C) $a = 12$ (D) $a = 14$
-

- 8.

| Score | Frequency |
|-------|-----------|
| 21 | 2 |
| 22 | 4 |
| 23 | 6 |
| 24 | 1 |
| 25 | 1 |

A score of 25 is added to this sample.

Which of these measures will change?

- (A) Range
 (B) Median
 (C) Mode
 (D) Mean
-

9. Simplify $9^3 \div 9^{-1}$.

- (A) 3^{-6} (B) 1 (C) 3^4 (D) 3^8
-

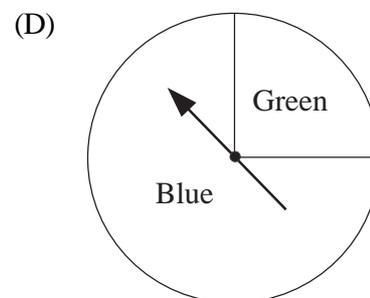
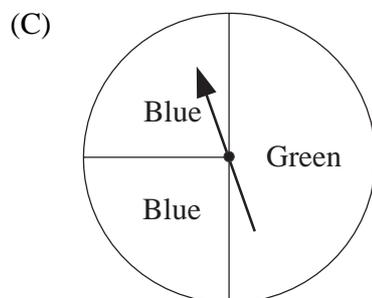
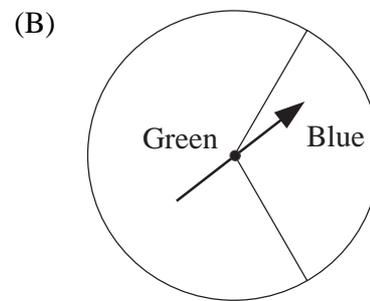
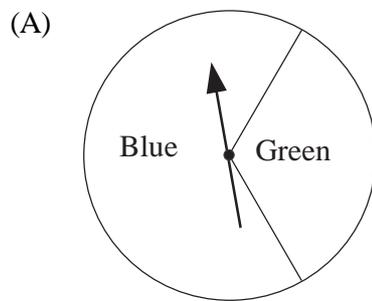
10. The equal sides of an isosceles triangle are each 2 cm longer than the third side. The third side has length x cm.

What is the perimeter of the triangle?

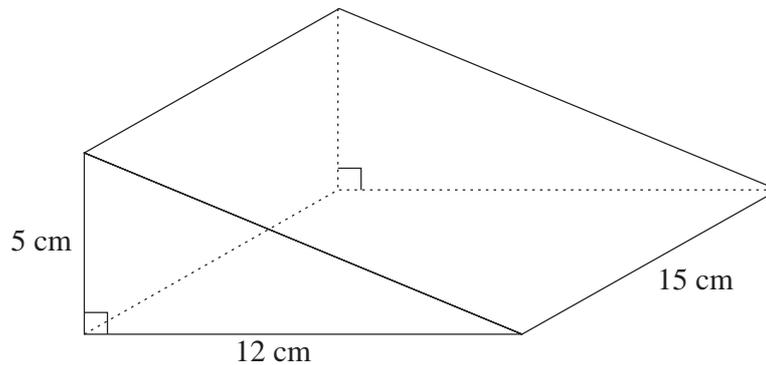
- (A) $(3x + 4)$ cm (B) $(3x - 2)$ cm (C) $(3x + 2)$ cm (D) $5x$ cm
-

11. A spinner is to be designed so that the probability of the arrow landing in the blue region is twice that of landing in the green region.

Which of the following designs meets this specification?



12. Calculate the surface area of this triangular prism.



- (A) 450 cm^2 (B) 510 cm^2 (C) 540 cm^2 (D) 570 cm^2
-

13. The stopping distance of a car is proportional to the square of the car's speed. A car travelling at 60 km/h has a stopping distance of 40 m.

If the stopping distance is 90 m, what is the car's speed?

- (A) 40 km/h (B) 90 km/h (C) 110 km/h (D) 135 km/h
-

14. Fifty tickets are sold in a raffle. There are two prizes. Michelle buys 5 tickets.

The probability that Michelle wins both prizes is given by the expression

- (A) $\frac{5}{50} + \frac{4}{50}$ (B) $\frac{5}{50} + \frac{4}{49}$ (C) $\frac{5}{50} \times \frac{4}{50}$ (D) $\frac{5}{50} \times \frac{4}{49}$
-

15. Seong Hye was required to solve an equation for homework. This is her solution.

$$5 - 3(2x - 4) = 20$$

$$5 - 6x + 12 = 20 \dots\dots\dots \text{Line 1}$$

$$-6x - 7 = 20 \dots\dots\dots \text{Line 2}$$

$$-6x = 27 \dots\dots\dots \text{Line 3}$$

$$x = -\frac{6}{27} \dots\dots\dots \text{Line 4}$$

Which lines *do not* follow correctly from the previous line?

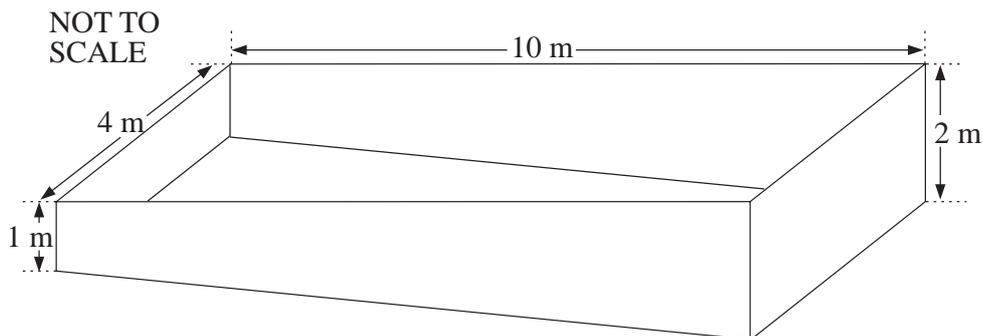
- (A) Line 1 and line 2 (B) Line 1 and line 4
 (C) Line 2 and line 3 (D) Line 2 and line 4
-

16. After five English tests, Sue's mean mark was 65. In the next three English tests she scored 70, 75, and 80.

Calculate Sue's mean mark for all of these English tests.

- (A) 68.75 (B) 70 (C) 72.5 (D) 75
-

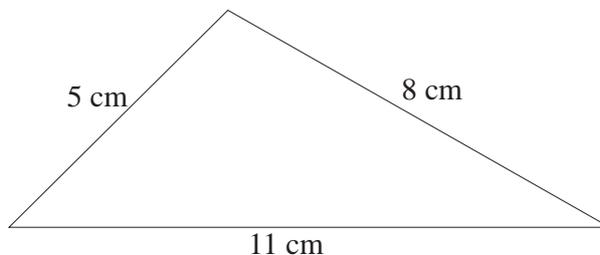
17. A swimming-pool has a length of 10 m and a width of 4 m. The depth of the pool is 1 m at one end and 2 m at the other end, as shown in the diagram.



What is the volume of the pool in cubic metres?

- (A) 40 (B) 60 (C) 80 (D) 120
-

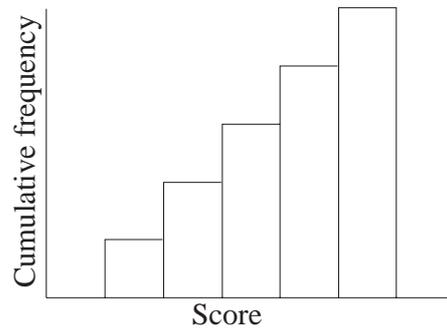
18. The largest angle in this triangle is θ .



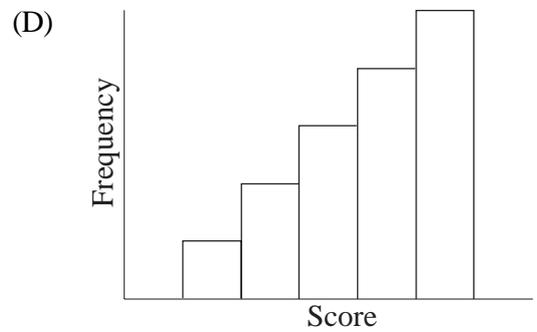
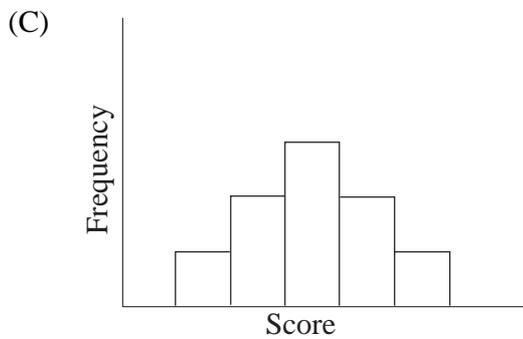
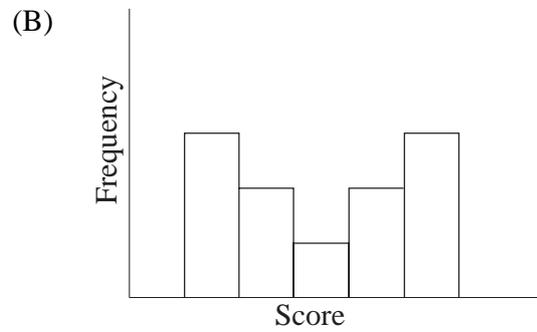
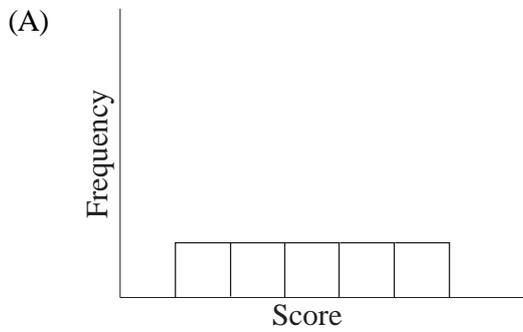
Which equation would give the correct value for θ ?

- (A) $\cos \theta = \frac{5^2 + 8^2 - 11^2}{2 \times 5 \times 8}$ (B) $\cos \theta = \frac{8^2 + 11^2 - 5^2}{2 \times 8 \times 11}$
- (C) $\cos \theta = \frac{5^2 + 11^2 - 8^2}{2 \times 5 \times 11}$ (D) $\cos \theta = \frac{5^2 + 8^2 - 11^2}{2 \times 8 \times 11}$
-

19. The diagram shows a cumulative frequency histogram.



For the same set of scores, which of the following best represents the frequency histogram?



20. A company changes the size of its muesli slice by decreasing all three dimensions by 5%.

The percentage decrease in volume is closest to

- (A) 5%. (B) 12.5%. (C) 14%. (D) 15%.
-

SECTION II

Attempt ALL questions.

Each question is worth 12 marks.

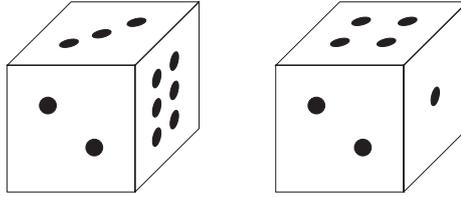
Show all necessary working.

QUESTION 21. Use a *separate* Writing Booklet.

- (a) 120% of a number is 75. What is the number?
- (b) An inflated balloon in the shape of a sphere has diameter 32 cm.
- What is its volume? $\left(V = \frac{4}{3}\pi r^3 \right)$
 - The balloon, while retaining its shape, has its volume increased to 25 000 cm³.
What is the diameter of the balloon now?
 - A cube also has a volume of 25 000 cm³.
What is the length of the side of this cube?
- (c) The ratio of heads to tails given by a biased coin is 2 : 3.
- What is the probability of obtaining a tail when this coin is tossed once?
 - What is the probability of obtaining at least one head when this coin is tossed twice?

QUESTION 21. (Continued)

(d)



Two ordinary dice are rolled and the difference between the two numbers appearing on the uppermost faces (larger number minus the smaller number) is noted. (For the dice shown above, the difference is $4 - 3 = 1$.)

The table below shows all the possible differences.

TABLE OF DICE DIFFERENCES

| | | <i>Dice 1</i> | | | | | |
|---------------|---|---------------|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 |
| <i>Dice 2</i> | 1 | 0 | 1 | 2 | 3 | 4 | 5 |
| | 2 | 1 | 0 | 1 | 2 | 3 | 4 |
| | 3 | 2 | 1 | 0 | 1 | 2 | 3 |
| | 4 | 3 | 2 | 1 | 0 | 1 | 2 |
| | 5 | 4 | 3 | 2 | 1 | 0 | 1 |
| | 6 | 5 | 4 | 3 | 2 | 1 | 0 |

- (i) What is the probability of obtaining a difference of 1?
- (ii) Mila and Nwong play a game of dice differences where each person begins with a score of 12. They take turns to roll the pair of dice and subtract the difference obtained each time from their score.

1. Nwong's first difference is 1. His score is then reduced to 11.

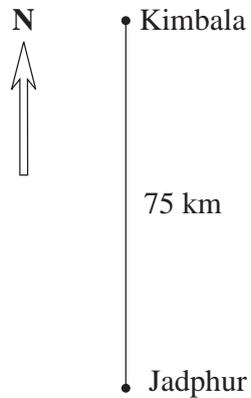
What is the minimum number of turns he will need to reduce his score from 11 to exactly 0?

2. Mila's first difference is 5, so his score is reduced to 7.

What is the probability that he will reduce his score to exactly 0 after *another* two turns?

QUESTION 22. Use a *separate* Writing Booklet.

- (a) Jadphur is a town 75 kilometres directly south of the town Kimbala, as shown on the diagram below.



The town Lati is 50 kilometres from Kimbala on a bearing of 125° .

- (i) Copy the above diagram into your Writing Booklet.

Using the above information, mark the approximate position of the town Lati on your diagram.

- (ii) Use the cosine rule to find the distance from the town Lati to the town Jadphur.

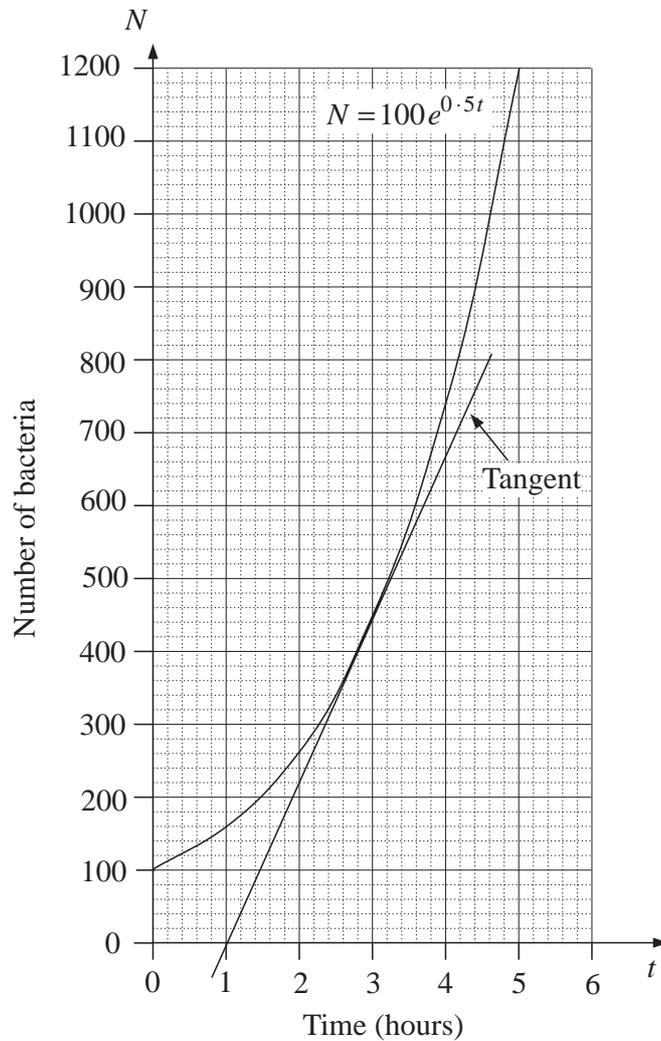
$$\left(\text{Cosine rule : } c^2 = a^2 + b^2 - 2ab \cos C \right)$$

QUESTION 22. (Continued)

(b) Bacteria are being cultivated in a dish.

After t hours, the number of bacteria (N) is given by $N = 100e^{0.5t}$.

A graph of this function is drawn below.

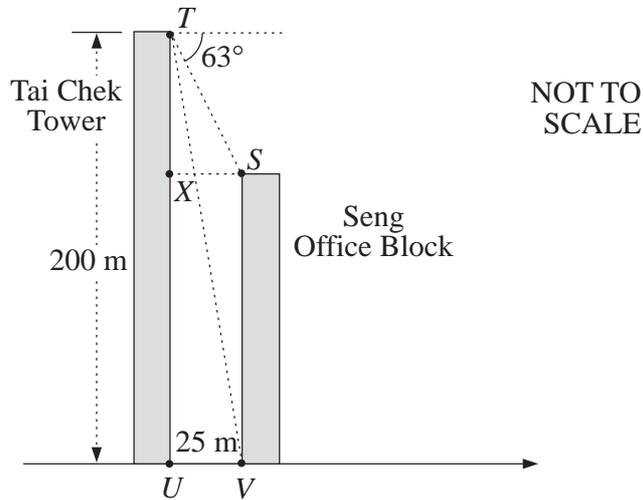


- (i) Use the graph to estimate the value of t when the number of bacteria will be approximately 1000. Give your answer in hours and minutes.
- (ii) The rate of change of the number of bacteria at $t = 3$ hours is equal to the slope of the tangent to the curve at $t = 3$. This tangent is shown on the above diagram.

Find the rate of change of the number of bacteria when $t = 3$.

QUESTION 22. (Continued)

- (c) Two buildings in Hong Kong are situated on level ground 25 m apart. Tai Chek Tower is 200 m high. The angle of depression from the top (T) of the Tai Chek Tower to the top (S) of the Seng Office Block is 63° . This is shown in the diagram below.



- (i) In your Writing Booklet, draw triangle TUV , marking in all relevant information.
Find the distance TV (correct to 1 m).
- (ii) Show that $TX = 49$ m (to the nearest metre) and hence write down the height of the Seng Office Block.
- (iii) Explain why $\angle TSV = 153^\circ$.
- (iv) Use the sine rule in $\triangle TSV$ to find the size of $\angle STV$.

$$\left(\text{Sine rule: } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \right)$$

- (v) What is the angle of depression from the top (T) of the Tai Chek Tower to the base (V) of the Seng Office Block?

QUESTION 23. Use a *separate* Writing Booklet.

(a) Solve the equation $4 - \frac{2}{x} = \frac{3}{2x}$.

(b) The following table shows statistics of the results of six mathematics tests for Jessie and Andrew.

| | <i>Mean</i> | <i>Standard deviation</i> | <i>Range</i> |
|--------|-------------|---------------------------|--------------|
| Jessie | 13.9 | 3.3 | 8 |
| Andrew | 13.8 | 2.7 | 8 |

Which student, Jessie or Andrew, has more consistent test results, and why?

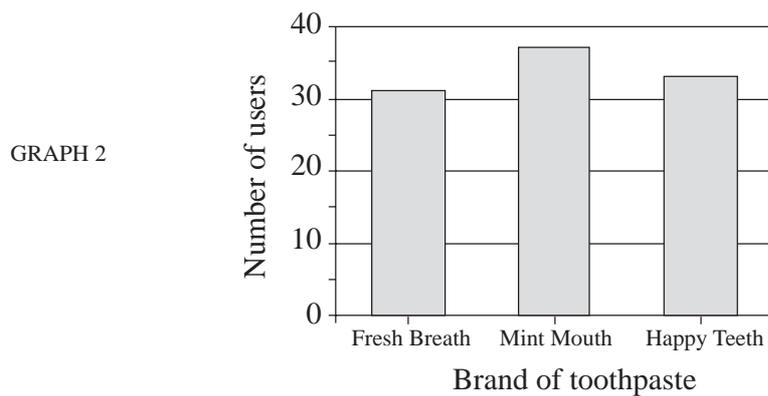
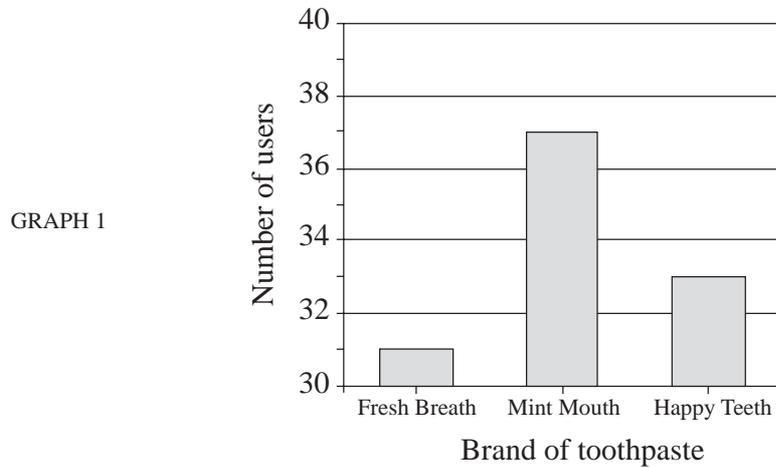
(c) A machine is set to produce metal disks with mean diameter 1.50 cm and standard deviation 0.01 cm. The diameters of these metal disks are normally distributed.

- (i) Within which interval will the mean diameter of the metal disks *almost certainly* lie?
- (ii) A metal disk produced by the machine is selected at random. This disk has a diameter of 1.54 cm.

Why does the machine operator then think that the machine has been set incorrectly?

QUESTION 23. (Continued)

- (d) Consumers were surveyed about the brand of toothpaste they used. The results of the survey are graphed in two different ways on Graph 1 and Graph 2 below.



- (i) Explain why the columns for Fresh Breath have different sizes in the two graphs.
- (ii) In a recent TV commercial the advertisers of Mint Mouth used Graph 1 instead of Graph 2.

Briefly explain why.

QUESTION 23. (Continued)

(e) A bag contains seven counters numbered as follows:



- (i) One counter is selected at random from the bag. What is the probability that there is an even number on it?
- (ii) Two counters are randomly selected from the bag without replacement.
 - 1. What is the probability that both counters have numbers less than 10 on them?
 - 2. Find the probability that the sum of the numbers on these two counters is less than 15.

SECTION III

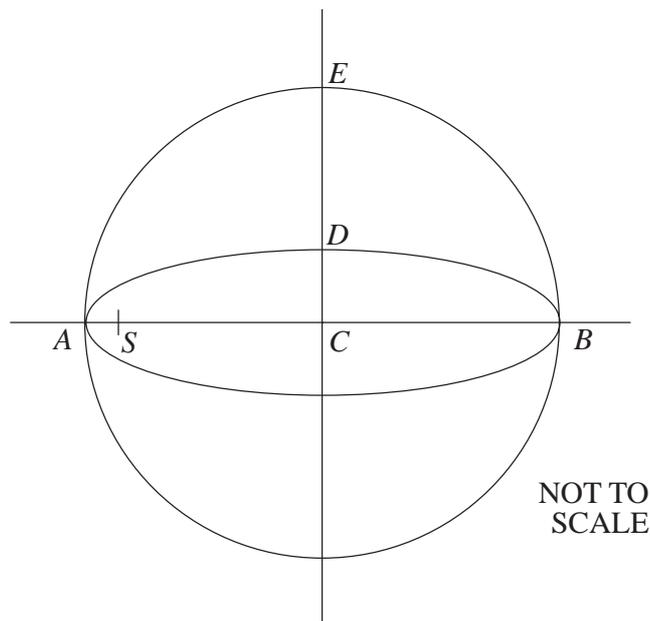
Attempt TWO questions.

Each question is worth 12 marks.

Show all necessary working.

QUESTION 24. Space Mathematics. Use a *separate* Writing Booklet.

(a) Halley's Comet moves in an elliptical orbit about the Sun (S) as shown.



The eccentricity of the orbit of Halley's Comet is 0.967. The average distance of Halley's Comet from the Sun is 17.96 astronomical units.

- (i) Use the formula $e^2 = 1 - \frac{b^2}{a^2}$ to calculate the ratio $\frac{b}{a}$ for the orbit of Halley's Comet.
- (ii) If BC is 17.96 astronomical units (AU), calculate the value of DC on the above diagram.
- (iii) Find, in kilometres, the distance represented by the length ED on the diagram.
(1 AU = $1.494\ 92 \times 10^8$ km)
- (iv) Find the length AS . ($CS = e \times CA$)
- (v) Given that R is the average distance from the comet to the Sun (in AU), and T is the period of the comet in years, use $R^3 = T^2$ to calculate the period of Halley's Comet in years.

QUESTION 24. (Continued)

- (b) A radio signal from Earth travels at the speed of light (3×10^8 m/s). It bounces off a satellite orbiting the Earth, taking 0.53 seconds from transmission to return.

Find the height of this satellite at the time of transmission.

- (c) (i) Define a 'light year'.

- (ii) The distance from Earth to a particular star is 8.19×10^{14} km.

Convert this distance to light years. (Speed of light = 3×10^5 km/s)

- (iii) A space probe from Earth travels at an average speed of 5.6×10^4 km/h.

Calculate the time, in years, that the space probe would take to travel the 8.19×10^{14} km to reach this star.

QUESTION 25. Mathematics of Chance and Gambling.Use a *separate* Writing Booklet.

- (a) Albert has 4 mathematics books and 1 physics book. He wishes to arrange these books on a shelf.

- (i) In how many different ways can the books be arranged?
- (ii) If the 4 mathematics books are to be next to one another, in how many different ways can these five books be arranged?

- (b) Twelve horses will run in a race.

- (i) To win a quinella, a punter must select the horses that will come first and second (in any order).

What is the probability of winning the quinella?

- (ii) The odds for one of the horses, Dufflecoat Supreme, are $3/1$ *on*. Gai bets \$120 on Dufflecoat Supreme.

How much will she collect if it wins?

- (c) George and Jerry book into separate rooms in a small hotel. In this hotel there are six rooms, arranged as shown in the diagram.

| | | |
|---|---|---|
| 1 | 2 | 3 |
| | | |
| 4 | 5 | 6 |

- (i) Jerry has booked room number 2. George's room is assigned randomly.

What is the probability that George is in a room adjoining Jerry's room?

- (ii) If both rooms are assigned randomly, what is the probability that George and Jerry will be in adjoining rooms?

QUESTION 25. (Continued)

- (d) A simple poker machine consists of three wheels. The ten letters shown below are on each wheel.

A, B, C, D, E, F, G, H, I, J

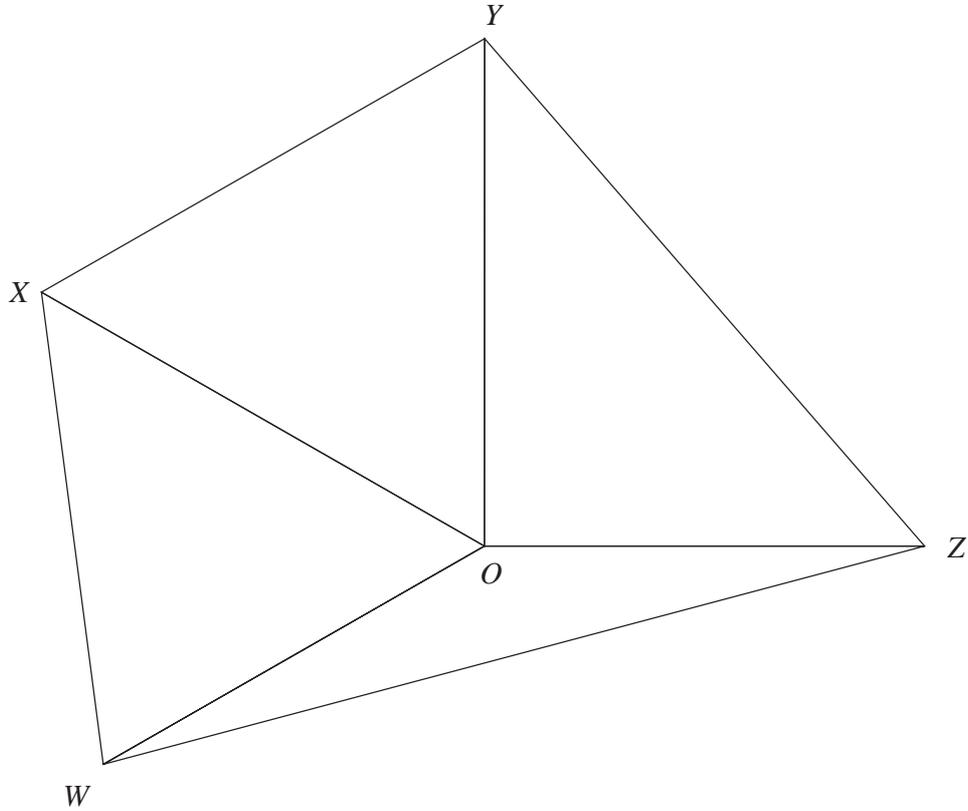
When the handle of the machine is pulled, the wheels spin, and the window on the front of the machine shows three letters in a row—one letter from each wheel. On each wheel the letters are equally likely to appear.

- (i) What is the probability that three J's will show in the window?
- (ii) In how many ways can the wheels stop so that just two letters in the window are J's, and the other letter is not a J?
- (iii) What is the probability that exactly two J's will show in the window?
- (iv) It costs \$1 to play the poker machine once. The pay-outs are as follows:
 - three J's showing: \$300;
 - exactly two J's showing: \$20.

What is the expected return on \$1000 from this poker machine?

QUESTION 26. Land and Time Measurement. Use a *separate* Writing Booklet.

- (a) A surveyor made the following scale drawing from a plane table survey.



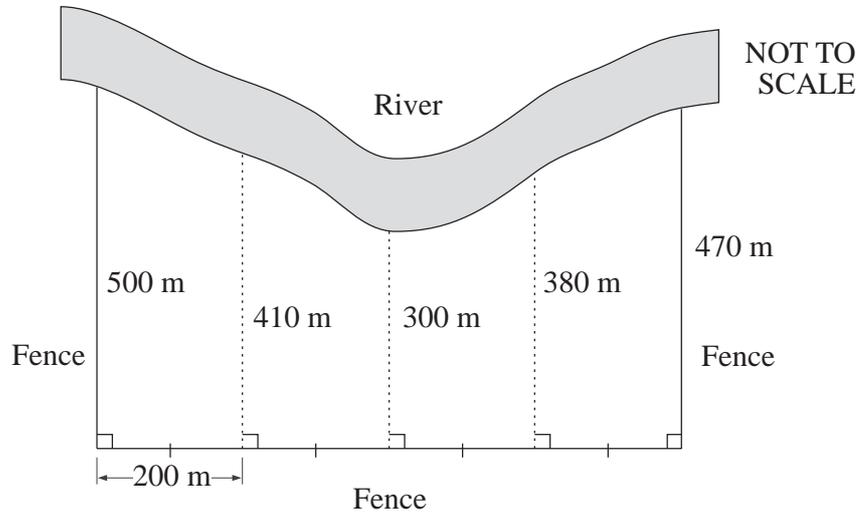
SCALE: 1 mm = 1 m

- (i) By measuring and using the scale, determine the lengths of OW and OX in metres.
- (ii) Use your protractor to measure $\angle XOW$ to the nearest degree.
- (iii) Find the area of $\triangle XOW$.

$$\left(\text{Area} = \frac{1}{2} ab \sin C \right)$$

QUESTION 26. (Continued)

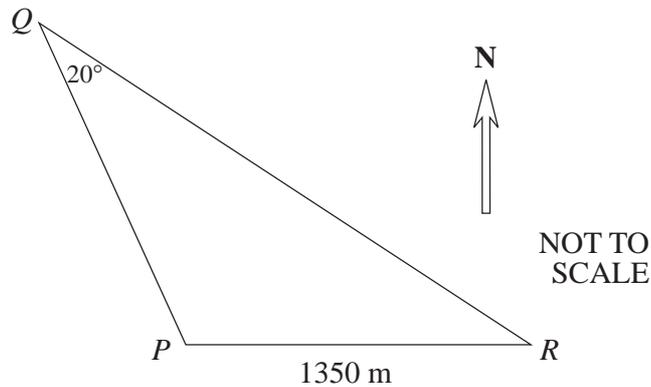
- (b) The diagram below shows an area of land bounded by a river and fences.



Use TWO applications of Simpson's rule to find the approximate area of this land.

$$\left(\text{Area} \approx \frac{h}{3}(d_F + d_L + 4d_M) \right)$$

- (c) A surveyor's diagram is shown below.



The bearing of Q from P is 342° and $\angle PQR = 20^\circ$. The distance from P to R is 1350 metres and R is due east of P .

- Find the size of $\angle QPR$.
- Use the sine rule to find the distance QR to the nearest metre.

$$\left(\text{Sine rule : } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \right)$$

QUESTION 26. (Continued)

- (d) The position of Carnarvon in Western Australia is 25°S , 114°E .
- (i) If it is midnight in Greenwich, what time should it be in Carnarvon (ignoring time zones)?
 - (ii) Find the radius of the 25°S circle of latitude to the nearest kilometre. (The radius of Earth is 6400 km.)
 - (iii) Bundaberg in Queensland has position 25°S , 152°E . Find the distance between Carnarvon and Bundaberg along the 25°S circle of latitude.

QUESTION 27. Personal Finance. Use a *separate* Writing Booklet and the graph paper on page 29.

(a) Janis' salary is currently \$38 000. She will receive salary increases as follows:

- 8% increase from 1 December 1997;
- a *further* 7% increase from 1 July 1998.

What will her salary be after 1 July 1998?

(b) A computer shop advertises the following computers for sale or rent.

Computer A
Purchase price \$1899
OR
Rent for \$21.10 per week

Computer B
Purchase price \$2475

(i) Hwee Min rents Computer A.

For how many weeks will she rent this computer before an amount equal to its purchase price has been paid?

(ii) The ratio of the weekly rental cost to purchase price is the same for Computer B as for Computer A.

Calculate the cost per week of renting Computer B .

(c) Jacob borrowed \$100 000 at an interest rate of 9.5% per annum. This rate was fixed for 3 years. He is to pay back only the interest over this period.

(i) How much interest is to be paid over the 3 years?

(ii) After paying the fixed rate of interest for 2 years, Jacob finds that the bank will drop his interest rate to 7.8% if he pays a charge of \$500.

How much will he save by changing to the lower interest rate for the last year?

QUESTION 27. (Continued)

- (d) The table below gives the personal income tax rates.

| <i>Taxable income</i> | <i>Tax payable</i> | <i>Tax bracket</i> |
|-----------------------|--|--------------------|
| \$0 – \$5 400 | Nil | <i>A</i> |
| \$5 401 – \$20 700 | \$0 plus 20c for each \$1 over \$5 400 | <i>B</i> |
| \$20 701 – \$38 000 | \$3 060 plus 34c for each \$1 over \$20 700 | <i>C</i> |
| \$38 001 – \$50 000 | \$8 942 plus 43c for each \$1 over \$38 000 | <i>D</i> |
| Over \$50 000 | \$14 102 plus 47c for each \$1 over \$50 000 | <i>E</i> |

- (i) Find the tax payable on a taxable income of \$60 000.
- (ii) The graph of the taxable income against tax payable has been started on page 29 of this paper.

Using the values in the table, and your answer in part (i), complete the graph.

- (iii) Rosemary pays \$11 400 in tax.

From your graph, read off the value of her taxable income.

- (iv) What percentage of her taxable income is paid as tax?

- (v) Stefan pays 20% of his taxable income in tax.

Which tax bracket (*A*, *B*, *C*, *D*, or *E* from the table) applies to his taxable income?

Note: Page 29 is to be detached, completed, and attached to your Writing Booklet.

QUESTION 28. Mathematics in Construction. Use a *separate* Writing Booklet.

- (a) The drawing below shows the floor plan of a three-bedroom house with a porch.

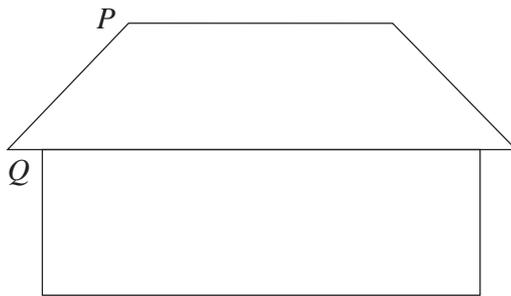
**DUE TO COPYRIGHT RESTRICTIONS THIS
IMAGE COULD NOT BE REPRODUCED.**

- (i) What scale is being used in the plan?
- (ii) Find the actual length from the front of the porch to the end of the kitchen (marked x on the plan).
- (iii) What is the total area of the house (including the porch)?
- (iv) The floor of Bedroom 1 is to be covered with square cork tiles. The tiles measure $30\text{ cm} \times 30\text{ cm}$. These tiles come in packets of 12, costing \$30 per packet.
- How much will it cost to purchase tiles for this area?
- (v) There is a problem with the design of the linen cupboard (L) in the laundry (L'DRY).

Using the plan, explain what this problem is (in one or two lines).

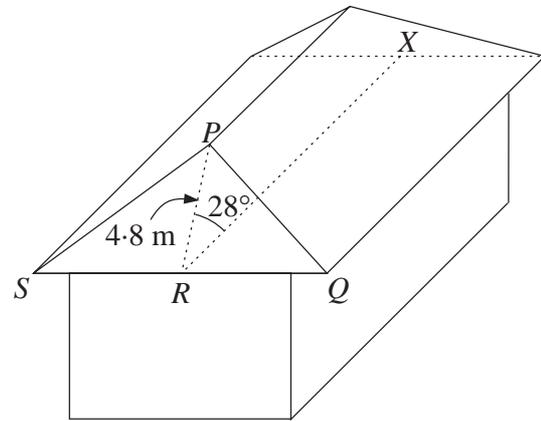
QUESTION 28. (Continued)

- (b) Figure 1 shows a sketch of the side elevation of a house. Figure 2 shows a sketch from the front of this house.



Side elevation

FIG. 1



Front

FIG. 2

- (i) What name is given to a roof with this shape?
- (ii) The distance PR from the guttering to the top of the roof is 4.8 m. The angle PRX is 28° .
Calculate the height of the point P above the line RX .
- (iii) If $PQ = PS = 6.6$ m, calculate the length SQ .

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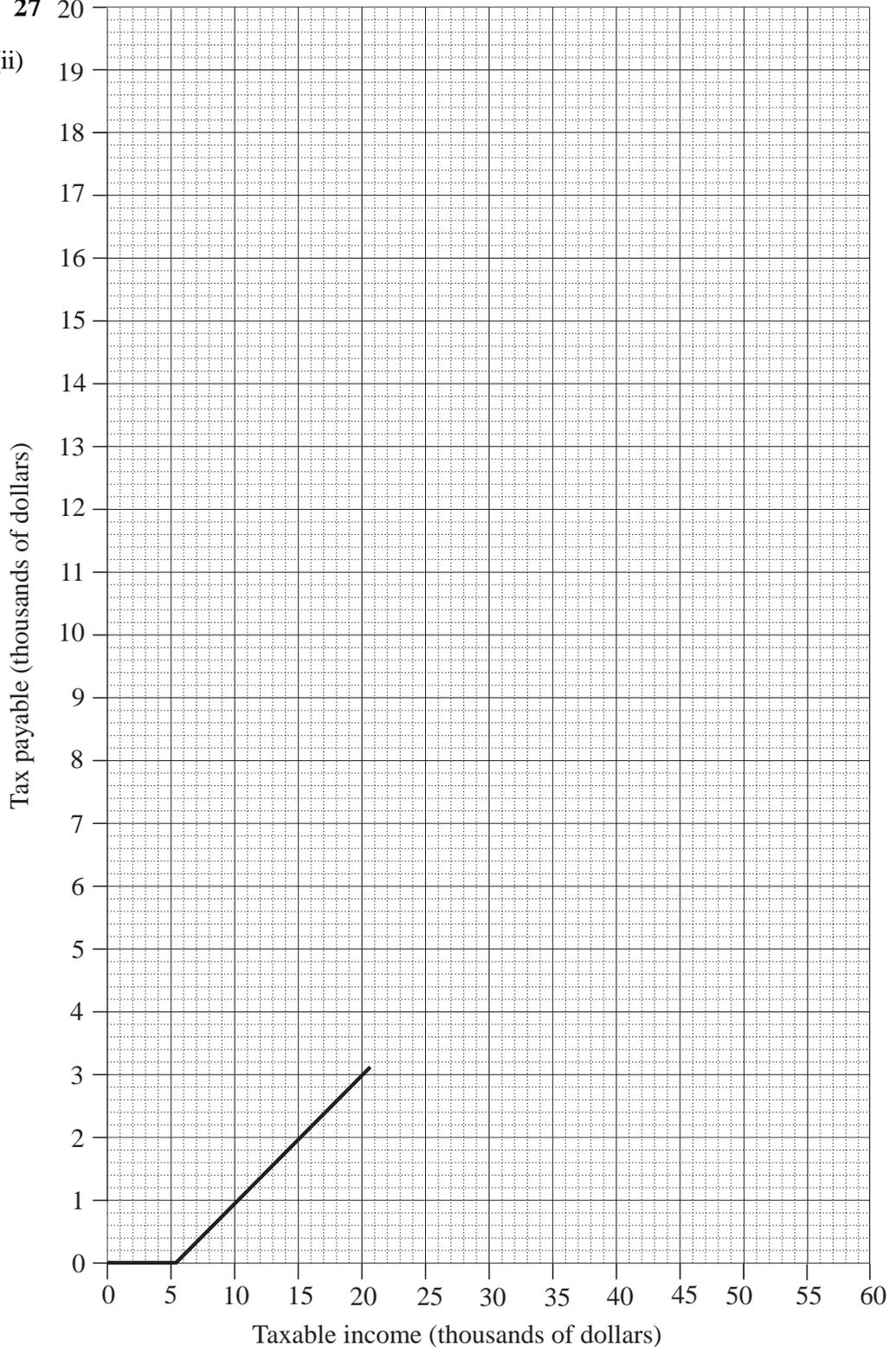
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If you are attempting Question 27, this page is to be detached, completed, and attached to your Writing Booklet for this question.

QUESTION 27

(d) (ii)



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