BUSINESS MATHEMATICS
(Three hours and a quarter)

(The first fifteen minutes of the examination are for reading the paper only. Candidates must NOT start writing during this time.)

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*Answer **Question 1** from Section **A** and **10 Questions** from Section **B**.*
All working, including rough work, should be done on the same sheet as, and adjacent to, the rest of the answer.

*The intended marks for the **Question** or parts of **Question** are given in brackets [ ]*
Mathematical formulae are given at the end of this Question paper.
The use of calculator (fx-82/fx-100) without memory is allowed.
Diagrams given in this question paper are not in scale.

Section A (30 Marks)
Answer **ALL** questions

Directions: Read the following questions carefully. For each question there are four alternatives A, B , C and D. Choose the correct alternative and write it in your answer sheet.

Question 1 **[2x15 = 30 Marks]**

i) The restrictions of $\frac{3x}{x+2}+\frac{x+5}{x-2}$ are

A $x \neq 2,\quad x \neq -2$

B $x \neq -2,\quad x \neq 0$

C $x \neq 2,\quad x \neq -5$

D $x \neq 3,\quad x \neq -5$

ii) If $y = x^4 + 4x$ then $\left.\frac{d^2y}{dx^2}\right|_{x=1}$ is

A 8

B 5

C 12

D 4

iii) If $\sin\theta = \frac{12}{13}$ and $\frac{\pi}{2} < \theta < \pi$, then the value of $\sin 2\theta$ is

A $\frac{120}{169}$

B $\frac{-120}{169}$

C $\frac{60}{169}$

D $\frac{-60}{169}$

iv) $\int \frac{dx}{3x+4}$ is

A $\log(3x+4)+c$

B $3\log(3x+4)+c$

C $\frac{1}{3\log(3x+4)}+c$

D $\frac{1}{3}\log(3x+4)+c$

v) The sigma notation of $1-1-3-5-7-9$ is

A $\sum_{i=1}^{6}(4i-3)$

B $\sum_{i=1}^{6}(5-4i)$

C $\sum_{i=1}^{6}(3-2i)$

D $\sum_{i=1}^{6}(2-i)$

vi) The adjoint of the matrix $\begin{pmatrix} 2 & 1 \\ 3 & -1 \end{pmatrix}$ is

A $\begin{pmatrix} -1 & -1 \\ -3 & 2 \end{pmatrix}$

B $\begin{pmatrix} -1 & -1 \\ 3 & 2 \end{pmatrix}$

C $\begin{pmatrix} 1 & 1 \\ -3 & -2 \end{pmatrix}$

D $\begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}$

vii) If $f(x) = \sqrt{2x+1}$, the value of $f'(x)$ is

A $2x\sqrt{2x+1}$

B $\dfrac{2x}{\sqrt{2x+1}}$

C $\dfrac{1}{2\sqrt{2x+1}}$

D $\dfrac{1}{\sqrt{2x+1}}$

viii) If $\sum_{i=1}^{100} a_i = 564$ and $\sum_{i=65}^{100} a_i = 263$, then the value of $\sum_{i=1}^{64} a_i$ is

A -301

B 301

C 564

D 827

ix) $\int \frac{\cos x - \sin x}{\cos x + \sin x} dx$ is

A $\log(\cos x + \sin x) + c$

B $\log(\cos x - \sin x) + c$

C $\cos x - \sin x + c$

D $\cos x + \sin x + c$

x) The trend value of the following data using a three year moving average is

Year	1	2	3	4	5	6	7
Values	2	4	6	8	10	12	14

A 2, 3, 4, 5, 6

B 8, 12, 16, 20, 24

C 4, 6, 8, 10, 12

D 12, 18, 24, 30, 36

xi) If $y = 5^{2x}$ then $\frac{dy}{dx}$ is

A $5^{2x} \log 5$

B $5^{2x} \log 25$

C $25 \log 5$

D $\frac{5^{2x}}{\log 25}$

xii) The derivative of $\log_e \sqrt{x}$ is

A $\frac{1}{\sqrt{x}}$

B $\frac{1}{2\sqrt{x}}$

C $\frac{1}{x^2}$

D $\frac{1}{2x}$

xiii) Simplified form of $\sqrt[3]{25} \times \sqrt[3]{10}$ is

A $5\sqrt[3]{10}$

B $5\sqrt{10}$

C $5\sqrt[3]{2}$

D $5\sqrt{2}$

xiv) If orange is sold for Nu. 40 per kilogram in the year 2007 and Nu. 50 per kilogram in the year 2008.Then, the price relative for the year 2008 is

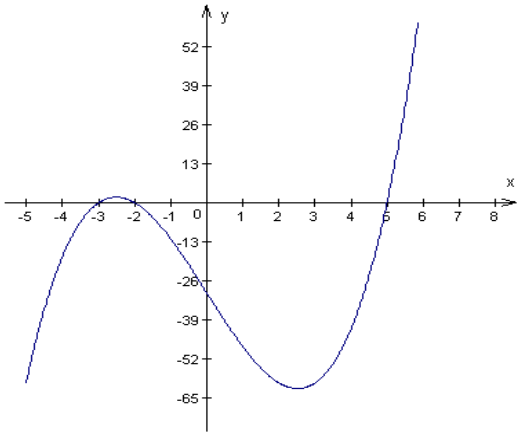
A 125

B 100

C 80

D 200

xv) The equation for the function for the given graph



A $y = -x^3 + 19x - 30$

B $y = x^3 + 10x^2 + 19x - 30$

C $y = x^3 + 10x^2 - 19x - 30$

D $y = x^3 - 19x - 30$

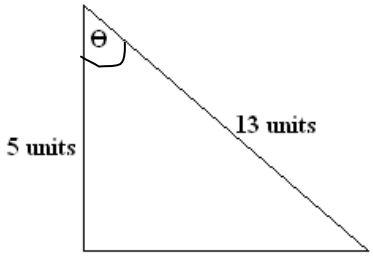
Section B (70 marks)

Answer any 10 questions. All questions in this section have equal marks. Unless otherwise stated, you may round answers to decimal places.

Question 2

a) If one root of $x^4 + 2x^3 + mx^2 - 18x = 0$ *is* -2. Determine the value of 'm' and state the equation. **[4]**

b) Using the diagram, evaluate $\sin\theta + \cos\theta$ **[3]**



Question 3

a) Find the value of $\sum_{i=1}^{20}(3i^2 - 4i + 1)$ **[3]**

b) Solve the inequality $x^3 + 24 > 3x^2 + 10x$ **[4]**

Question 4

a) Simplify and state the restrictions $\dfrac{1}{x^2 - 6x + 9} - \dfrac{1}{x^2 - 3x}$ **[3]**

b) Determine the equation of the tangent line to $y = -5x^4 + 2x^3 - 7x + 3 \quad at \quad x = 1$ **[4]**

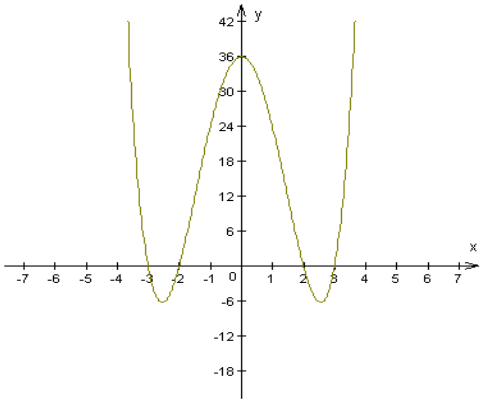
Question 5

a) Using induction method, prove that $\sum_{i=1}^{n}\dfrac{1}{i(i+1)} = \dfrac{n}{n+1}, \quad n \in N$ **[5]**

b) Find $\dfrac{dy}{dx} \quad if \quad y = \left(x^2 + 2\right)^3$ **[2]**

Question 6

a) For the given graph, find **[4]**



i) x- intercept
ii) y- intercept
iii) maxima and minima

b) Differentiate $y = x^3.e^{2x}$ *w.r.t.* $'x'$ **[3]**

Question 7

a) An apple orchard having 100 trees produces an average of 200 apples per tree. The owner is expanding the orchard with 20 trees every year and due to improved farming methods the annual crop is increasing at the rate of 10 apples per tree. **[4]**

i) Write an equation to represent the annual production, 'P', as a function of 'n', the number of year from now.

ii) Determine the current rate of increase in the annual production.

b) Find $\frac{dy}{dx}$ *for* $y = \cos^2(4x)$ **[3]**

Question 8

a) Karma invested Nu. 60,000 in Bhutan National Bank, the interest earned for the money invested is 6% p.a. compounded semi-annually. **[4]**

i) How much money will be in his account after 6 years?

ii) After how many years will the amount grow to Nu. 100,000?

b) If $x^2 + y^2 = 25$, find the value of $\frac{dy}{dx}$ at the point $(3,\ 4)$ **[3]**

Question 9

a) Find all the asymptotes of the function $f(x) = \dfrac{x}{x^2 - 3x + 2}$ **[4]**

b) Find the volume of the solid generated when the region under $y = x$, bounded between $x = 0$ and $x = 1$ is rotated about x-axis. **[3]**

Question 10

a) Find the derivative of $y = \dfrac{xe^x}{\log_e x}$ **[4]**

b) Show that $\tan\left(\dfrac{\pi}{4} - x\right) = \dfrac{\cos x - \sin x}{\cos x + \sin x}$ **[3]**

Question 11

a) Using matrix method, solve the system of simultaneous equations. **[4]**

$2x - 3y = 1$ *and* $5y = 7 - x$

b) Find $\int xe^x dx$ **[3]**

Question 12

a) Find the cost of living index from the data provided below. **[4]**

Group	Group Index	Weight
Food	130	10
Rent	152	45
Clothing	110	17
Fuel	105	13
Misc	85	15

b) Solve the equation **[3]**

$\sqrt{2x+5} = 2\sqrt{2x} + 1$

Question 13

a) Find the inverse of $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$ **[4]**

b) Find $\int (2x^2 - x + 5)\, dx$ **[3]**

Question 14

a) Find $\int_0^{\pi} (\cos x + \sin x)\, dx$ **[3]**

b) The marks of the students in English and Dzongkha are represented in the data below. **[4]**

English	30	33	45	23	8	49	12	4	31
Dzongkha	35	23	47	17	10	43	9	6	25

Calculate the Rank Correlation Co-efficient using Spearman's method.

Functions and Equations

(1) $(a \pm b)^2 = a^2 + b^2 \pm 2ab$

(2) $(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$

(3) $a^2 - b^2 = (a+b)(a-b)$

(4) $a^3 \pm b^3 = (a \pm b)\left(a^2 \mp ab + b^2\right)$

(5) $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Sequence and series

(1) $\sum_{i=1}^{n} i = \dfrac{n(n+1)}{2}$

(2) $\sum_{i=1}^{n} i^2 = \dfrac{n(n+1)(2n+1)}{6}$

(3) $\sum_{i=1}^{n} i^3 = \left[\dfrac{n(n+1)}{2}\right]^2$

(4) $t_n = ar^{n-1}$

(5) $t_n = a + (n-1)d.$

(6) $S_n = \dfrac{a\left(1-r^n\right)}{1-r}$ where $r<1$

$= \dfrac{a\left(r^n - 1\right)}{r-1}$, Where $r > 1$

(7) $S_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$

Differentiation

(1) $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

(2) $y = x^n,\ y' = nx^{n-1},$

(3) $y = cf(x),\ y' = cf'(x)$

(4) $y = f(x) \pm g(x),\ y' = f'(x) \pm g'(x)$

(5) $F(x) = f(x)g(x),$

$F'(x) = f(x)g'(x) + f'(x)g(x)$

(6) $F(x) = \dfrac{f(x)}{g(x)}$,

$$F'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{\left[g(x)\right]^2}$$

(7) $f \circ g(x)' = f'g(x) \times (g'x)$

(8) $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

(9) $v(t) = h'(t)$

Coordinate Geometry

(1) $(y - y_1) = m(x - x_1)$

(2) $\sqrt{(x-a)^2 + (y-b)^2}$

Trigonometry

(1) $Sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

(2) $Cos(A \pm B) = CosA \cos B \mp \sin A \sin B$

(3) $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

(4) $\sin^2\theta + \cos^2\theta = 1$

Logarithmic Exponentials

(1) $y = y_0(1+r)^x$

(2) $y = y_0 e^{kx}$

(3) $A = P(1+r)^n$

Integration

(1) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[\left(\dfrac{d}{dx}f(x)\right)\int g(x)dx\right]dx$

(2) $\int_a^b f(x)dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\Delta x$

(3) $V = \pi\int_a^b y^2 dx$

(4) $A = \int_a^b y dx$

Measurement

(1) Cone: $V = \dfrac{\pi}{3}r^2 h$

(2) Cone: $SA = \pi rl + \pi r^2$

(3) Sphere: $V = \dfrac{4\pi}{3}r^3$

(4) Sphere: $SA = 4\pi r^2$

(5) Cylinder: $SA = 2\pi rh + 2\pi r^2$

(6) Cylinder: $V = \pi r^2 h$

(7) Circle: $A = \pi r^2$

(8) Circle: $C = 2\pi r$

(9) Triangle: $A = \frac{bh}{2}$, $A = \frac{\sqrt{3}}{4}x^2$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

(10) Rectangle: $A = lw$,

(11) Rectangle $P = 2l + 2w$

(12) Square: $A = s^2$,

(13) Square $P = 4s$

(14) Rectangular Prism: $V = lwh$

Matrices

(1) $C_{ij} = (-1)^{i+j} M_{ij}$

(2) $A A^{-1} = A^{-1} A = I$

(3) *Inverse of* $A = A^{-1} = \frac{1}{\det A} \cdot adjA$

Data & Probability

(1) $\bar{x} = \frac{\sum fx}{n}$

(2) Median $= l_1 + \frac{l_2 - l_1}{f1}(m - c)$

(3) $\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$

(4) $\sigma_{12} = \sqrt{\frac{n_1\sigma_1^2 + n_1\sigma_2^2 + n_1 d_1^2 + n_2 d_2^2}{n_1 + n_2}}$

(5) $\sigma = \sqrt{\frac{\sum f(x_i - \bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{N} - \left(\frac{\sum fx}{N}\right)^2}$

(6) $\bar{x}_{12} = \frac{m\bar{x}_1 + n\bar{x}_2}{m+n}$

(7) $I = \frac{\sum \frac{P_1}{P_0} \times 100}{n}$

(8) $I = \frac{\sum p_1 w}{\sum p_0 w} \times 100$

(9) $\mathrm{Cov(X,Y)} = \frac{1}{n}\sum (X - \bar{X})(Y - \bar{Y})$

(10) $r = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2 \sum (y - \bar{y})^2}}$

(11) $r = \frac{\sum (x - \bar{x})(y - \bar{y})}{n\sigma_x \sigma_y}$

(12) $b_{YX} = \frac{\mathrm{cov(X,Y)}}{\sigma_x^2} = r\frac{\sigma_y}{\sigma_x}$

(13) $Y - \bar{Y} = \frac{\mathrm{cov}(X,Y)}{\sigma_x^2}(X - \bar{X})$

$$= r\frac{\sigma_x}{\sigma_y}(X - \bar{X})$$

(14) $b_{xy} \times b_{yx} = r\frac{\sigma_y}{\sigma_x} \times r\frac{\sigma_x}{\sigma_y}$

(15) $\tau = \frac{2S}{n(n-1)}$

(16) $r = 1 - \frac{6\sum d^2}{n(n^2 - 1)}$